



Some BASICS

Voltage, Current, Components and AC behavior,
Bode Plots, Transfer Functions, Thévenin Equivalent,
High-pass and Low-pass filters,...



Prefixes for Units

- For writing down small or large quantities, exponents can be used: $1.5 \times 10^6 \Omega$, $3 \times 10^{-9} A$
- To simplify, **prefixes** in steps of 1000 are used:
 - T Tera $\times 10^{12}$
 - G Giga $\times 10^9$
 - M Mega $\times 10^6$
 - k Kilo $\times 10^3$
 - 1 $\times 10^0$
 - m Milli $\times 10^{-3}$
 - μ (or u) Mikro $\times 10^{-6}$
 - n Nano $\times 10^{-9}$
 - p Piko $\times 10^{-12}$
 - f Femto $\times 10^{-15}$
 - a Atto $\times 10^{-18}$
- Try to learn: ‘Piko \times Kilo = Nano, Milli \times Mega = Kilo,...’

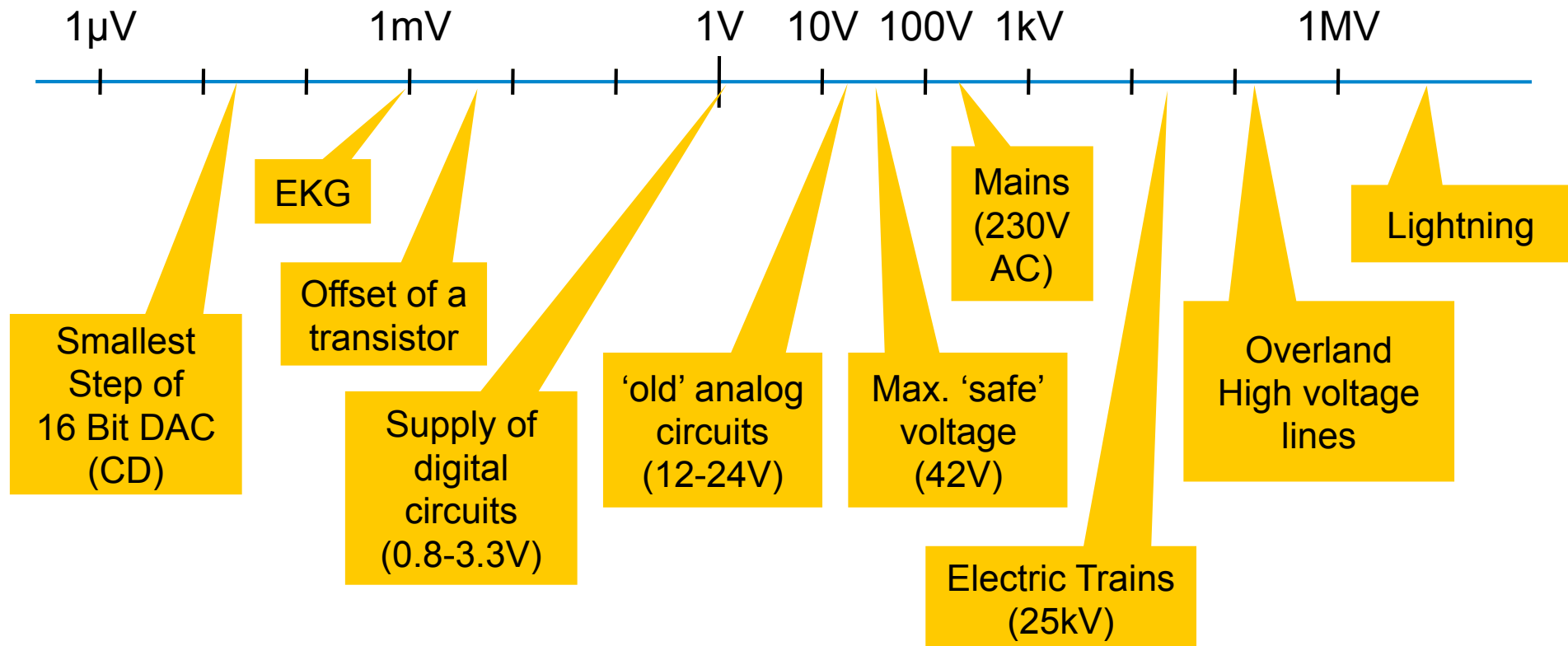


VOLTAGE, CURRENT, KIRCHHOFF'S LAWS



Voltage

- Voltage is the *difference* in electrical potentials, i.e. the energy required to move a unit charge in an electric field
 - This is only well defined in static fields where $\text{rot } \vec{E} = 0$
- Unit: Volt (V)





Ground

- Voltages are really potential **differences**
- To simplify life, we define a **reference potential** to which voltages are referred. We call it '**ground**'
 - i.e. when we say 'net A has 3V', we mean $V_A - V_{\text{GND}} = 3\text{V}$
 - Ground is at 0V by definition

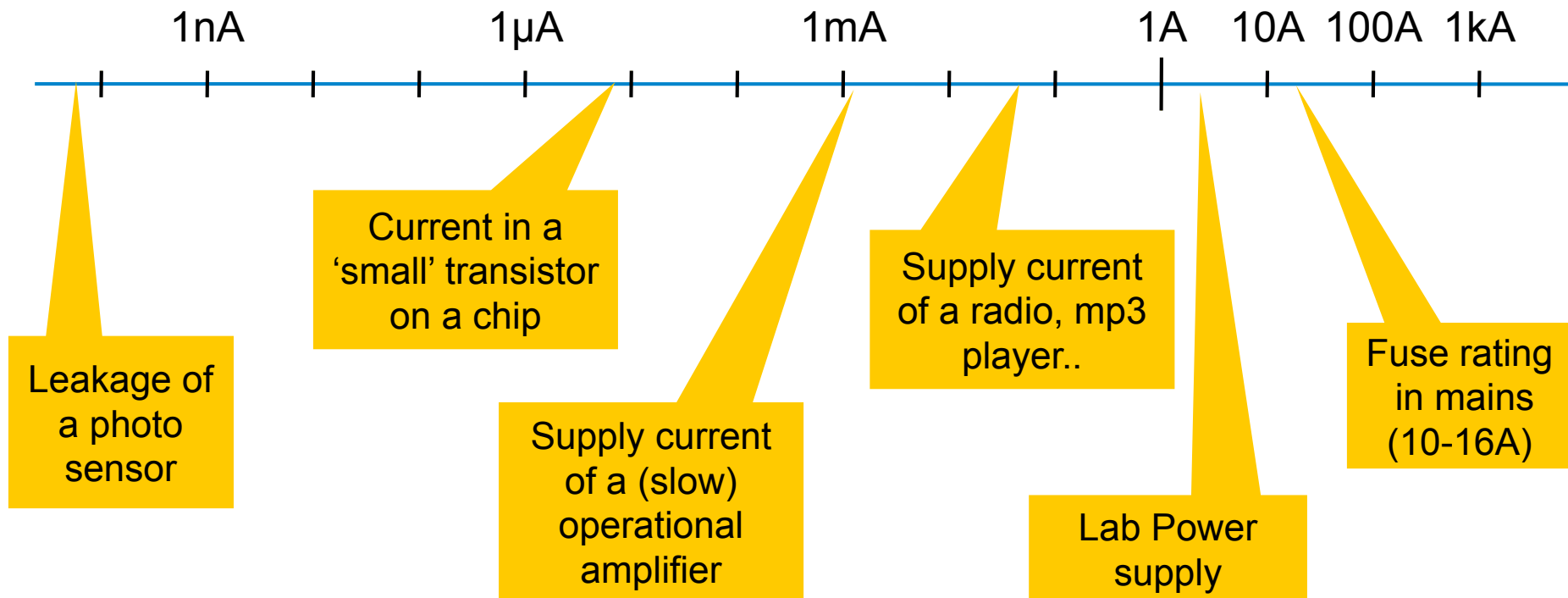
- Common ground symbol are: 

- (Later we may use several grounds, all at 0V, but separated, for digital and analogue circuit parts)



Current

- Electric current is the flow (or change) of electric charge
- $i = dQ / dt$
- Unit: Ampere (A)

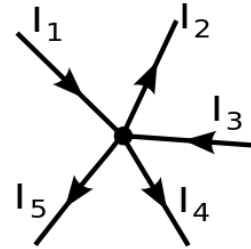




Kirchhoff's Laws

1. The sum of currents at any node is zero:

$$\sum_{k=1}^n I_k = 0$$



- Follows from charge conservation

2. The sum of voltages in any closed loop is zero:

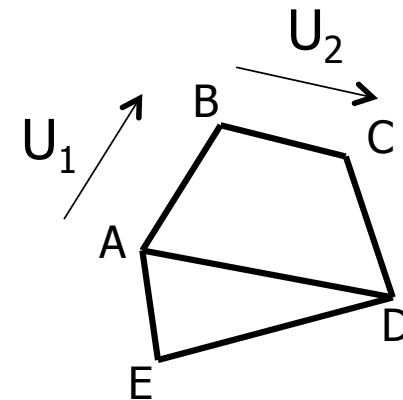
$$\sum_{k=1}^n U_k = 0$$

The sign of the U_k is fixed by a consistent ordering of the nodes in the loop.

Example:

$$U_1 = U_B - U_A, U_2 = U_C - U_B, \dots$$

$$U_1 + U_2 + U_3 + U_4 = 0$$



- Follows from energy conservation



RESISTORS & CAPACITORS



Resistors

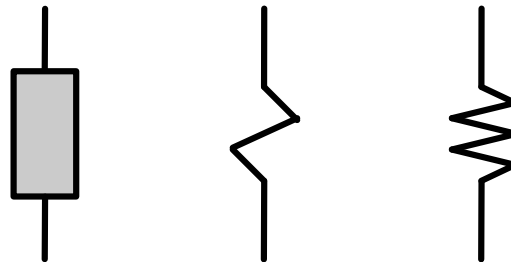
- A resistor is a 2 terminal device
- When voltage is applied, a current flows
- The current is **proportional** to the voltage (Ohms's law):

$I = U \times G$ G is the **conductivity** (Leitwert) in Siemens [S]
or

$I = U / R$ R is the **resistivity** (Widerstand) in Ohm [Ω]

- G and R describe the same relation. $G = 1/R$, $R = 1/G$

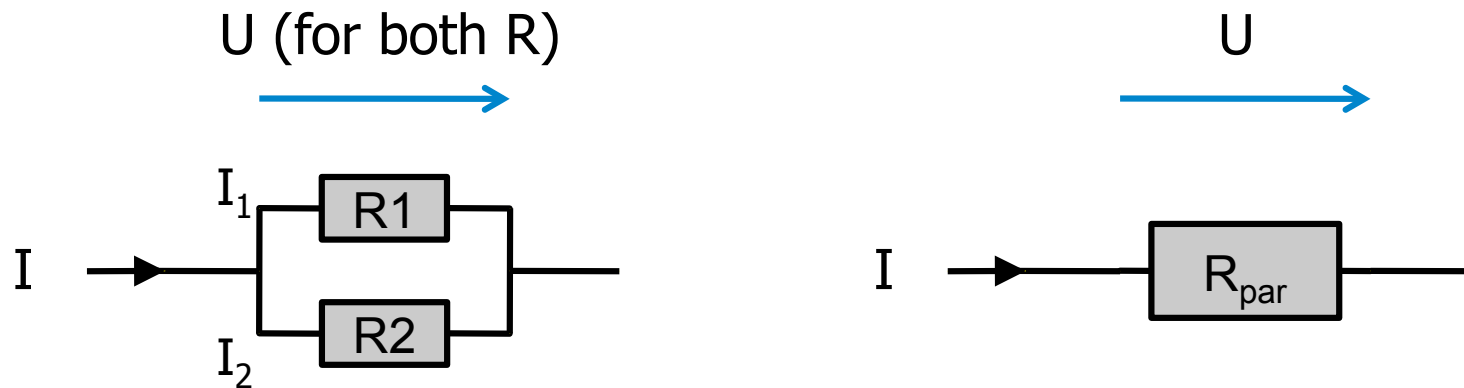
- Symbols:



- Note: **Ohm's law is not trivial**. Not all materials are 'ohmic'



Parallel Connection of Resistors



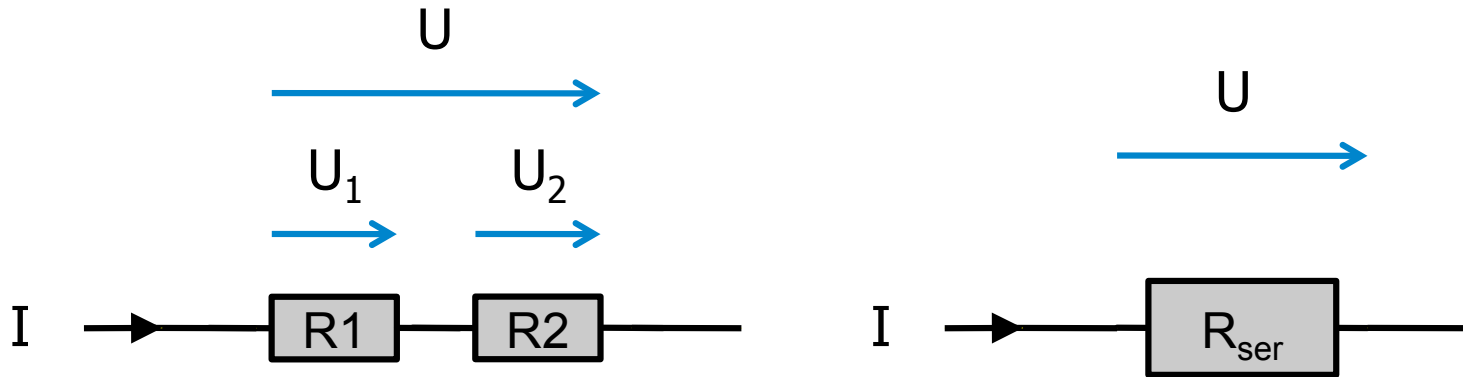
$$I = I_1 + I_2 = G_1 \times U + G_2 \times U = (G_1 + G_2) \times U$$

$$I = G_{\text{par}} \times U$$

$$G_{\text{par}} = G_1 + G_2 \quad \leftrightarrow \quad 1/R_{\text{par}} = 1/R_1 + 1/R_2$$



Series Connection of Resistors



$$U = U_1 + U_2 = I \times R_1 + I \times R_2 = I \times (R_1 + R_2)$$

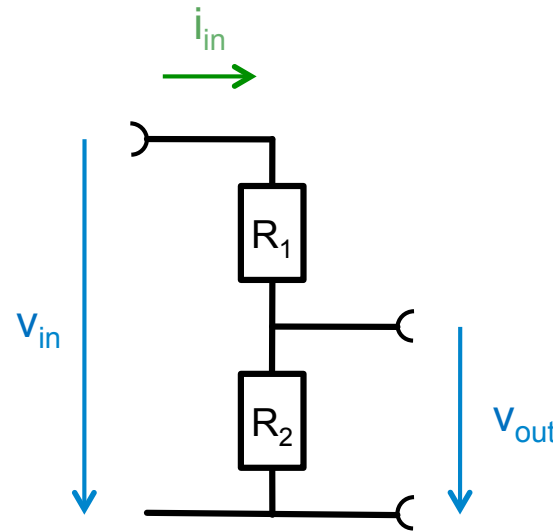
$$U = I \times R_{\text{ser}}$$

$$R_{\text{ser}} = R_1 + R_2 \quad \leftrightarrow \quad 1/G_{\text{ser}} = 1/G_1 + 1/G_2$$



The Voltage Divider (*without* load current!)

- A very common topology is the voltage divider:

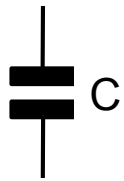


- The input current $i_{in} = v_{in} / (R_1 + R_2)$
- This current flows through R_1 and R_2 , i.e. $i_{in} = i_{R1} = i_{R2}$
- On R_2 , it develops a voltage $v_{out} = i_{R2} R_2 = v_{in} R_2 / (R_1 + R_2)$

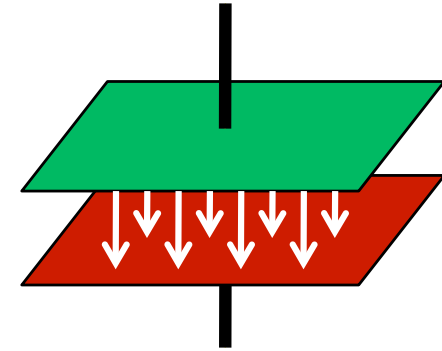
- Overall: $v_{out} / v_{in} = R_2 / (R_1 + R_2)$



Capacitors



- A capacitor can store electrical charge
- Prototype: parallel plate capacitor
 - **Charge Q** on plates generates field (through Gauss' law)
 - Field between plates gives a **voltage V**
- $Q = C \times V$: capacitance is factor between charge and voltage
 - A **large** capacitor can store a **lot** of charge at **low** voltage
- The voltage on a capacitor is given by the current integral:



$$V = \frac{Q}{C} = \frac{1}{C} \int I(t) dt \quad \Leftrightarrow \quad I(t) = C \frac{dV}{dt}$$

- The stored energy is:

$$dE(Q) = V(Q)dQ \Rightarrow E = \int_0^Q V(Q')dQ' = \int_0^Q \frac{Q'}{C}dQ' = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$



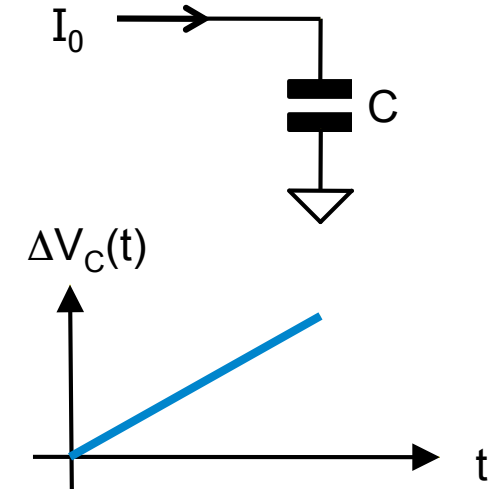
Charging a Capacitor

- At constant current I : **linear ramp**:

$$I(t) = I_0 = \text{const}$$

$$\Delta Q(t) = \int_0^t I(t') dt' = \int_0^t I_0 dt = I_0 \times t$$

$$\Delta U(t) = \frac{\Delta Q(t)}{C} = \frac{I_0}{C} \times t$$

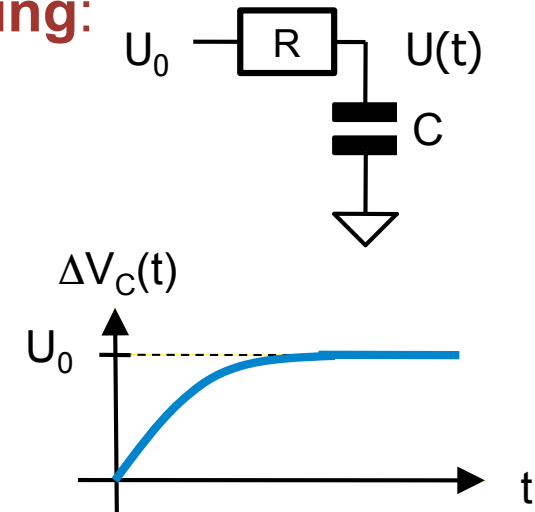


- Through resistor R : **exponential settling**:

$$I(t) = \frac{U_0 - U(t)}{R}$$

$$\frac{dU(t)}{dt} = \frac{I(t)}{C} = \frac{U_0 - U(t)}{RC}$$

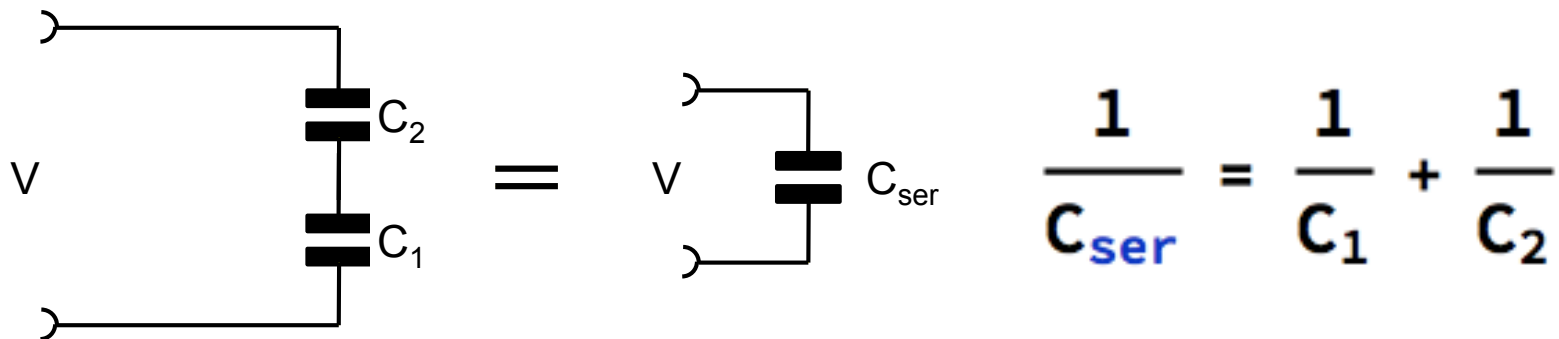
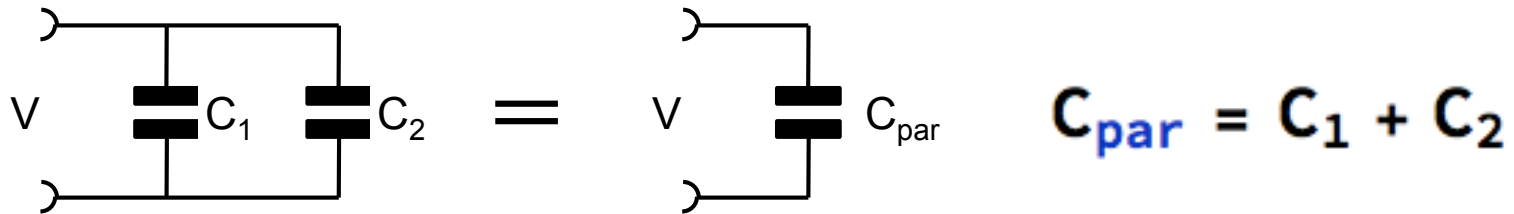
$$\text{Solution : } U(t) = U_0 - U_0 e^{-\frac{t}{RC}}$$





Parallel and Series Connection of Capacitors

- For derivation, see exercise...



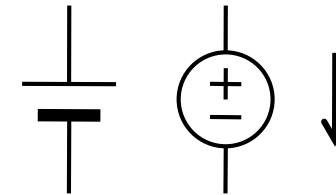


VOLTAGE & CURRENT SOURCES

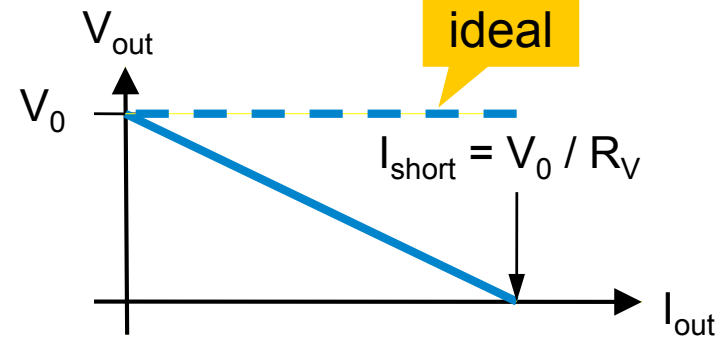
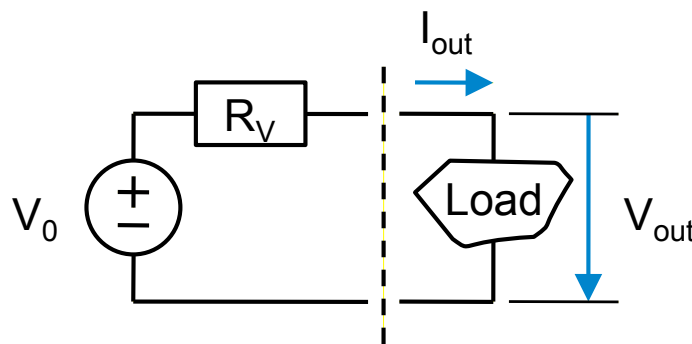


Voltage Sources

- A voltage source has 2 terminals:



- An **ideal** voltage source maintains the voltage for **any** output current ('1000 A')
- The voltage of a **real** source drops with **load current**.
- This is modeled by a **series** resistor (internal resistor, source resistor):

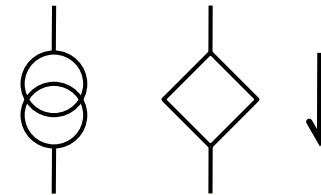


- The **open voltage** is V_0 ($I_{out}=0 \rightarrow$ voltage drop over R_V is 0)
- The **short circuit current** is $I_{short} = V_0 / R_V$
- Note: '**Good**' voltage sources have **low** $R_V \rightarrow 0$

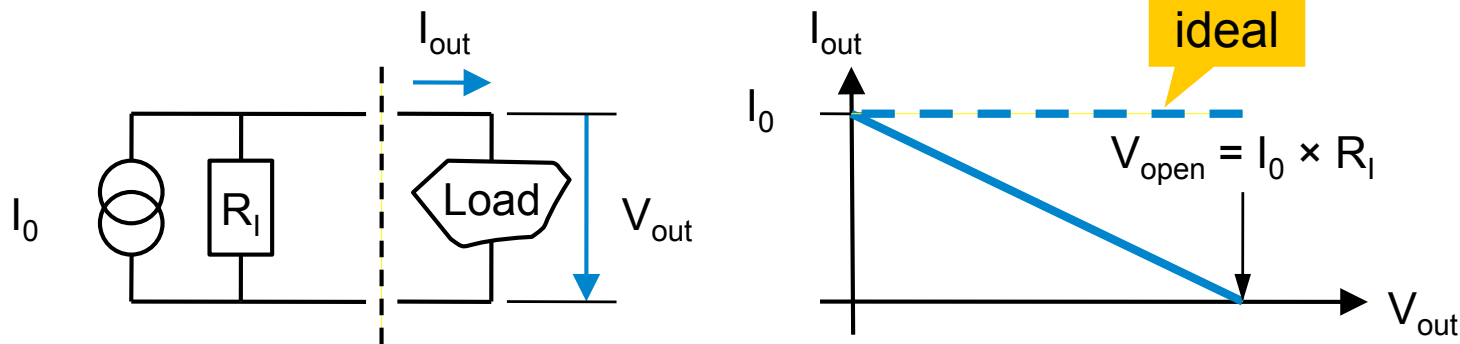


Current Sources

- A current source has 2 terminals:



- An **ideal** current source maintains the current for **any** output voltage
- The current of a **real** source drops with **load voltage**.
- This is modeled by a **parallel** resistor (internal resistor, source resistor):

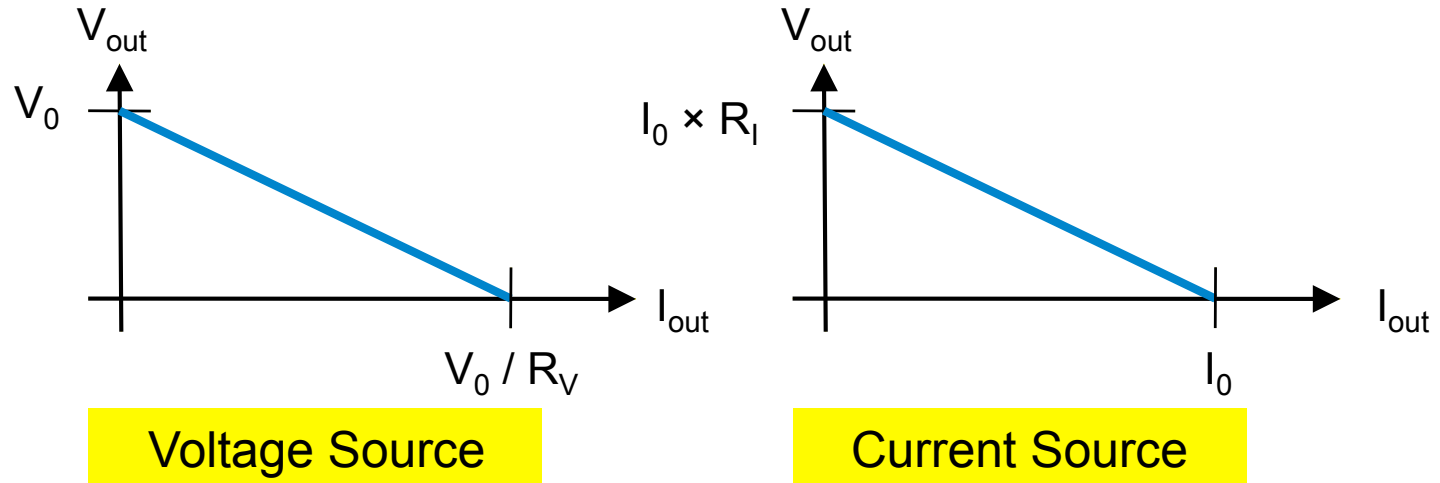


- The **short circuit current** is I_0 (no voltage at $R_1 \rightarrow$ no current)
- At a voltage of $I_0 \times R_1$ no more current flows (all flows in R_1)
- Note: '**Good**' current sources have **high** $R_1 \rightarrow \infty$

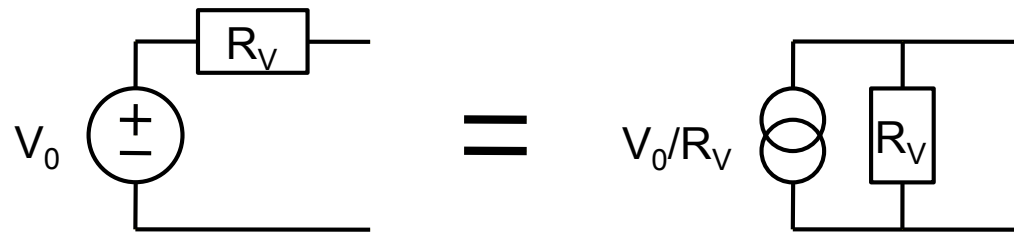


Equivalence of U- and I-Source

- Flip the diagram of the I-source and compare:



- Same shape! Therefore:
- For voltage source with V_0 and R_V , a current source with $I_0 = V_0 / R_V$ and $R_I = V_0 / I_0 = R_V$ behaves the same!



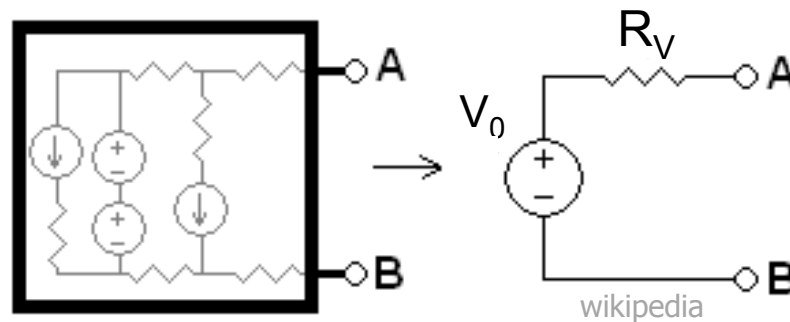


Thévenin's Theorem

Any combination of U-sources, I-sources and resistors behaves like a (real) voltage source with an internal resistor

- This is fairly obvious from the previous page and the linearity of the resistor properties
- Clearly, a current source with internal resistor can also be used

Example:

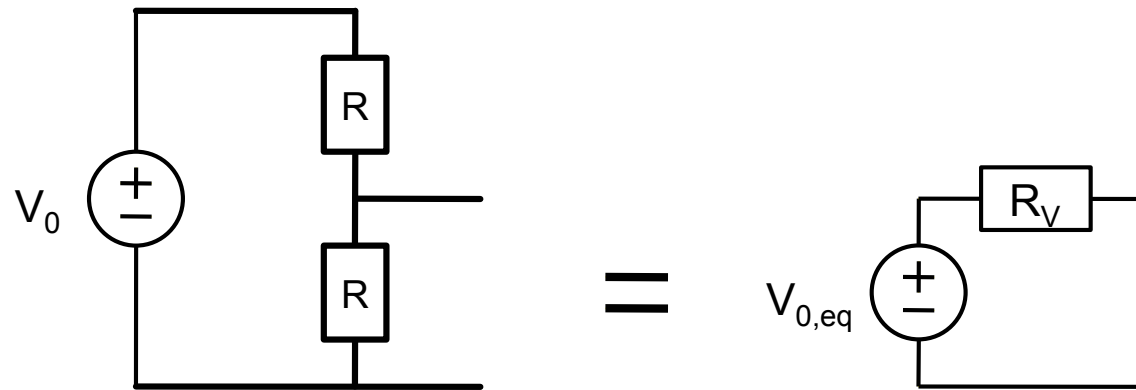


- To find V_0 : calculate the open voltage
- To find R_V : find the short circuit current. Then $R_V = V_0 / I_{\text{short}}$



Thévenin Equivalent of a Voltage Divider

- Consider a voltage divider with equal resistors:

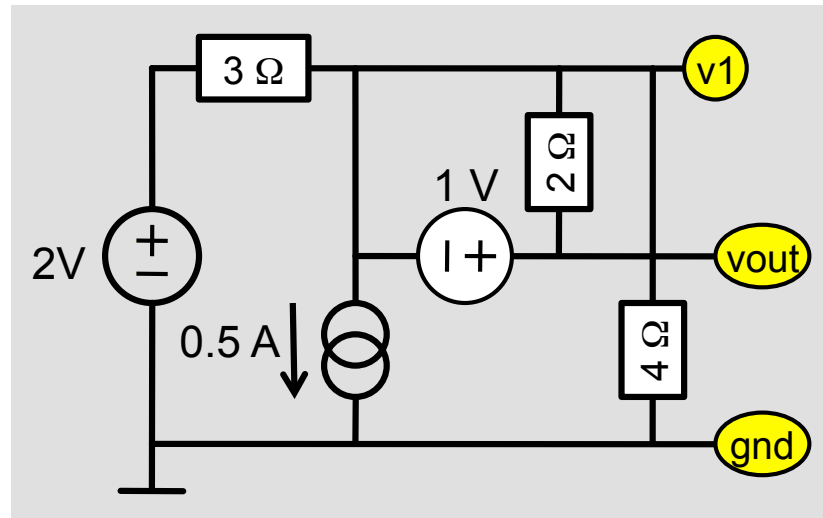


- $V_{0,eq} = V_0 / 2$ ($I = V_0 / (2R)$, $V_{0,eq} = R \times I$)
- $I_{short} = V_0 / R \rightarrow R_V = V_{0,eq} / I_{short} = R / 2$

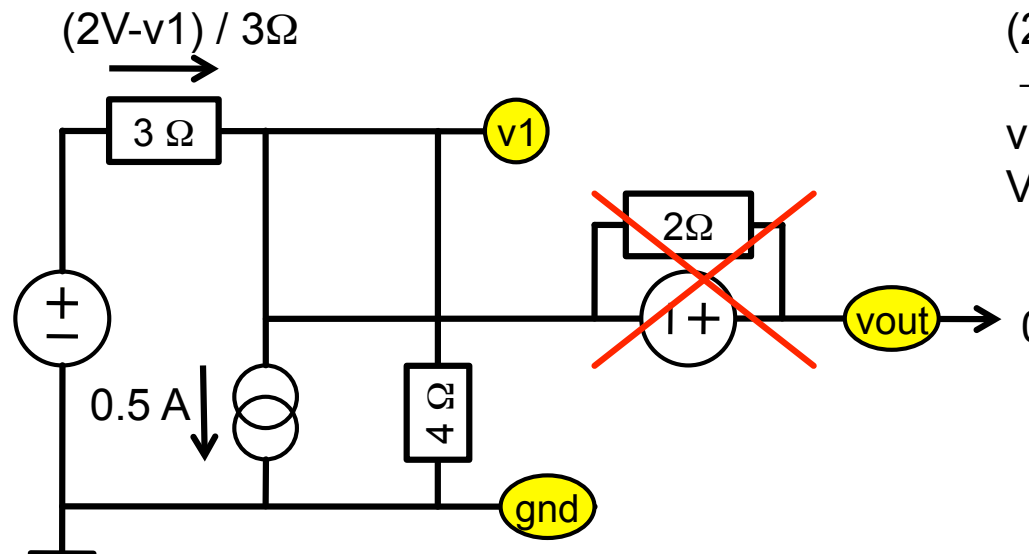
- In general, R_V is the parallel connection of R_1 and R_2



A More Complicated Example - 1



1. Open circuit:



$$(2V - v1) / 3\Omega = 0.5A + v1 / 4\Omega$$

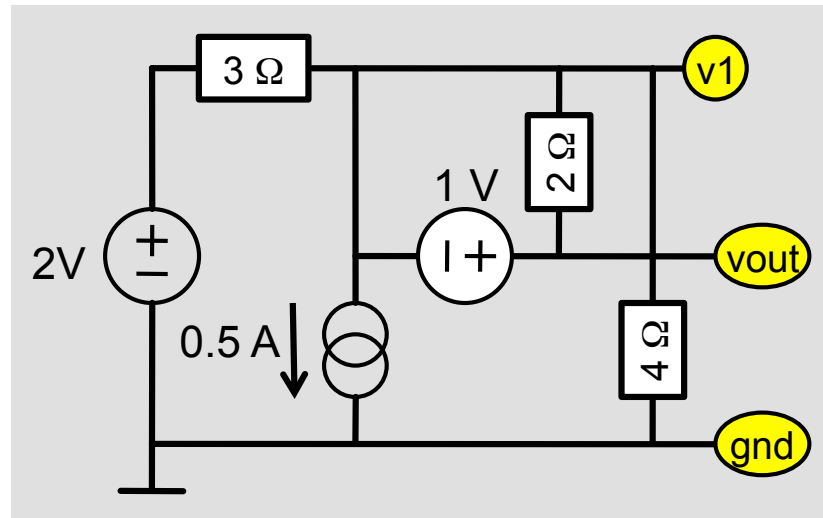
$$\rightarrow$$

$$v1 = 0.285714 V$$

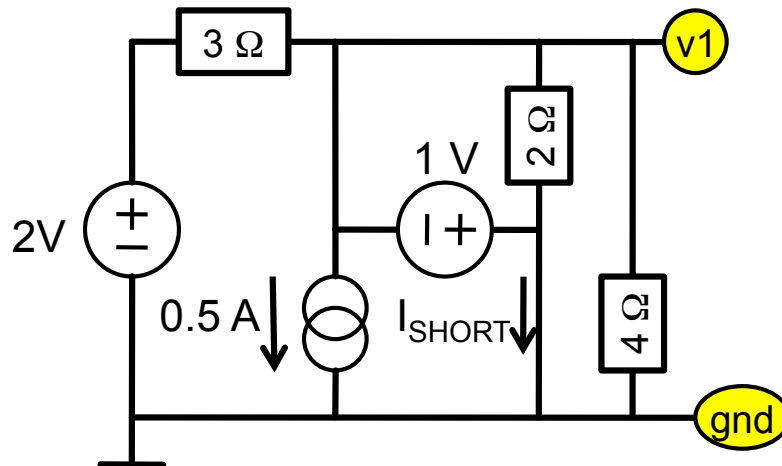
$$V_{out} = v1 + 1V = \mathbf{1.285 V = V_0}$$



A More Complicated Example - 2



1. Short circuit:



$$(2V - v_1) / 3\Omega = 0.5A + v_1 / 4\Omega + I_{SHORT}$$

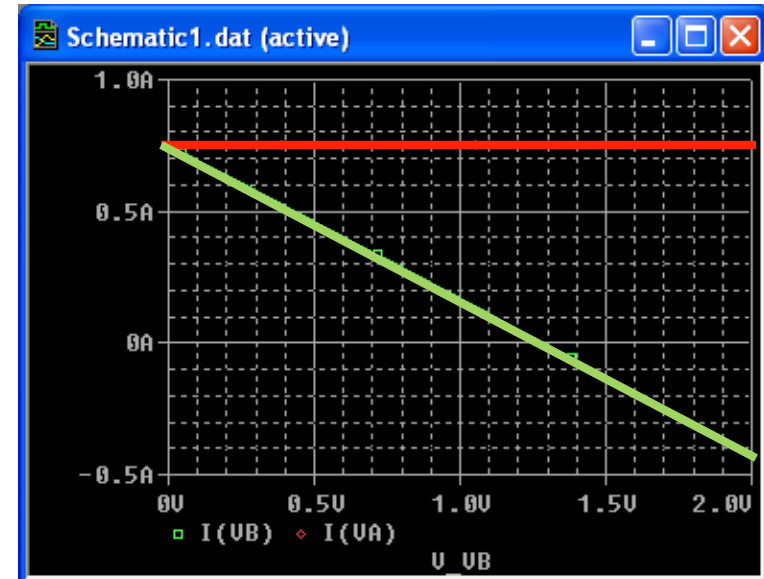
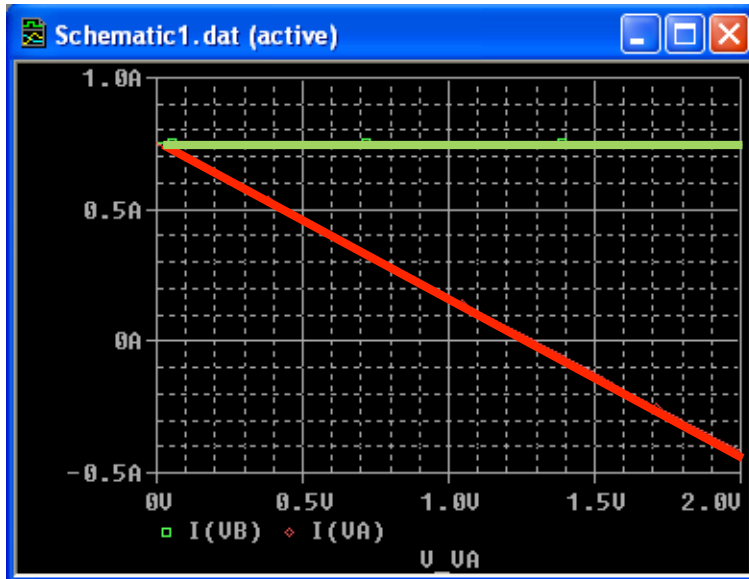
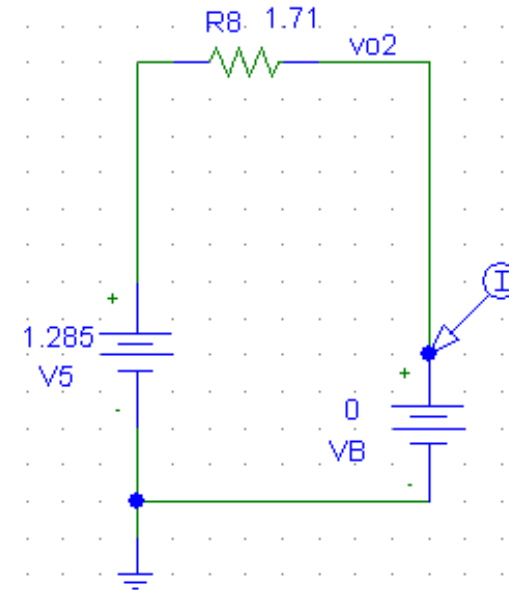
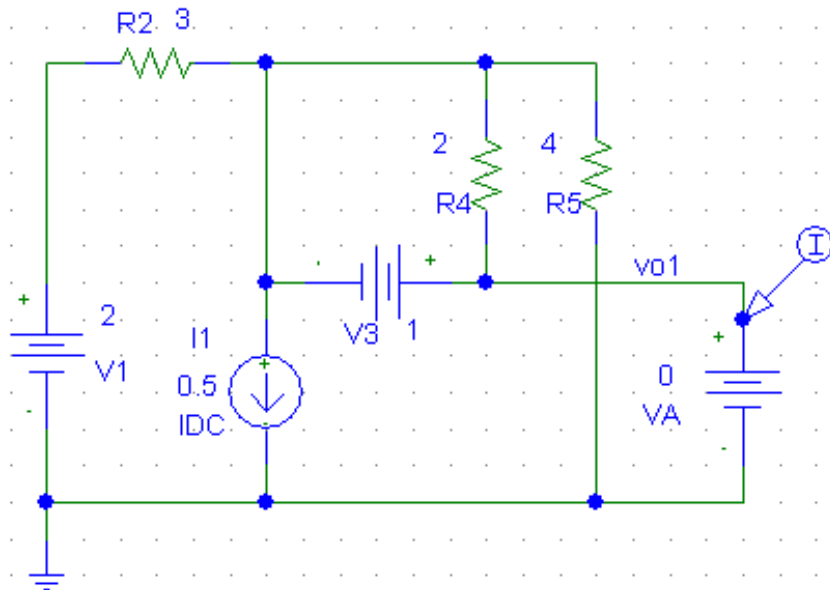
$$v_1 = -1V$$

$$\rightarrow I_{SHORT} = 0.75 A$$

$$R_V = V_0 / I_{SHORT} = 1.285 V / 0.75 A = 1.71\Omega$$



A More Complicated Example - Simulation



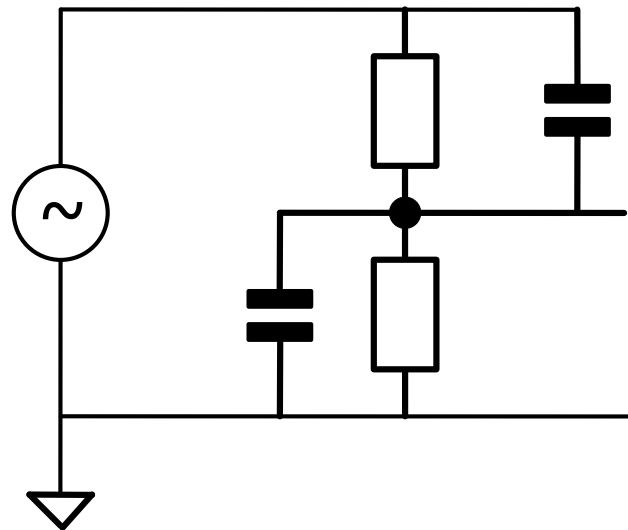
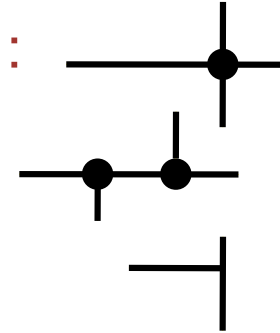


DRAWING SCHEMATICS



Drawing Schematics: Some Rules

- **Positive** voltages are at the **top**, negative at the bottom
- **Input** signals are at the **left**, **outputs** at the **right**
- Connected **crossings** are marked with a ● :
 - should be avoided
- T-connections do not need a ● :
 - but they can have one...





Example

- A useless circuit...

