CCS: Solutions of Excercise 01

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Some Default settings

```
SetOptions[{LogLinearPlot, LogLogPlot, Plot},
  Frame → True,
  GridLines → Automatic,
  Filling → Axis,
  MeshFunctions → {Log[#1] &},
  PlotRange → Full,
  ImageSize → 400,
  PlotStyle → Thickness[0.005],
  LabelStyle → Directive[FontSize → 12, FontFamily → "Helvetica"]
 ];
```

```
dB[x_] := 20 Log[10, x]; (* definition of decibel *)
```

```
dBGain[x_] := 10 Log[10, x Conjugate[x]] // Simplify
```

Exercise 1.2 (Power in Load resistor)

$$Vout = V0 \frac{RL}{R0 + RL};$$

$$Iout = \frac{Vout}{RL}$$

$$\frac{V0}{R0 + RL}$$

```
Pout = Vout Iout
```

$$\frac{RL\ V0^2}{(R0 + RL)^2}$$

```
Table[Limit[Pout, RL → x], {x, {0, ∞}}]
```

{0, 0}

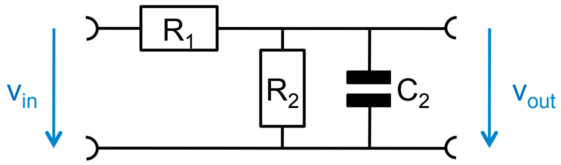
```
Solve[D[Pout, RL] == 0, RL] // First
```

{RL → R0}

```
Pout /. %
```

$$\frac{V0^2}{4\ R0}$$

Exercise 1.3 (modified Low Pass)



Direct Treatment:

$$\text{EQ} = \frac{\text{Vin} - \text{Vout}}{\text{R1}} == \text{Vout s C2} + \frac{\text{Vout}}{\text{R2}};$$

```
Solve[EQ, Vout] // First
```

$$\left\{\text{Vout} \to \frac{\text{R2 Vin}}{\text{R1} + \text{R2} + \text{C2 R1 R2 s}}\right\}$$

$$\text{Hdirect} = \frac{\text{Vout}}{\text{Vin}} \text{ /. } \%$$

$$\frac{\text{R2}}{\text{R1} + \text{R2} + \text{C2 R1 R2 s}}$$

Voltage Divider:

$$\text{Hdiv} = \frac{\text{Z2}}{\text{Z1} + \text{Z2}} \text{ /. } \left\{\text{Z1} \to \text{R1}, \text{Z2} \to \left(\frac{1}{\text{R2}} + \text{s C2}\right)^{-1}\right\} \text{ // Simplify}$$

$$\frac{\text{R2}}{\text{R1} + \text{R2} + \text{C2 R1 R2 s}}$$

```
Hdirect == Hdiv
```

True

Thenevin:

$$\text{Hthenevin} = \frac{\text{g}}{1 + \text{s RR C2}} \text{ /. } \left\{\text{g} \to \frac{\text{R2}}{\text{R1} + \text{R2}}, \text{RR} \to \left(\frac{1}{\text{R1}} + \frac{1}{\text{R2}}\right)^{-1}\right\} \text{ // Simplify}$$

$$\frac{\text{R2}}{\text{R1} + \text{R2} + \text{C2 R1 R2 s}}$$

```
Hdirect == Hthenevin
```

True

Plot

```
$Assumptions = ω > 0 && R1 > 0 && R2 > 0 && C2 > 0 && RC > 0;
```

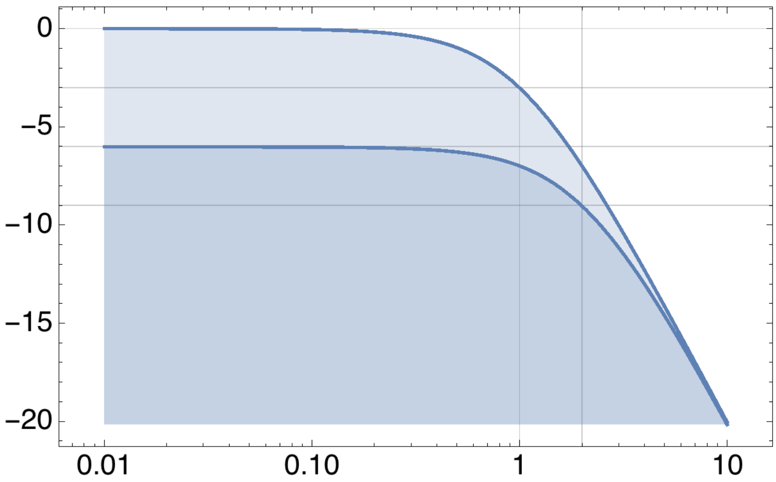
```
HLP = Sqrt[1/(1 + ⅈ ω RC) Conjugate[1/(1 + ⅈ ω RC)]] /. {RC → 1} // Simplify
```

$$\frac{1}{\sqrt{1+\omega^2}}$$

```
HExercise = Sqrt[(Hdirect /. s → ⅈ ω) Conjugate[Hdirect /. s → ⅈ ω]] /.
  {R1 → 1, R2 → 1, C2 → 1} // FullSimplify
```

$$\frac{1}{\sqrt{4+\omega^2}}$$

```
LogLinearPlot[dB[{HLP, HExercise}], {ω, 0.01, 10},
 GridLines → {{1, 2}, {0, -6, -3, -9}}, ImageSize → 400]
```



Exercise 1.4 (Notch Filter)

```
$Assumptions = ω > 0 && RC > 0; HLP = 1/(1 + ⅈ ω RC); HHP = (ⅈ ω RC)/(1 + ⅈ ω RC);
```

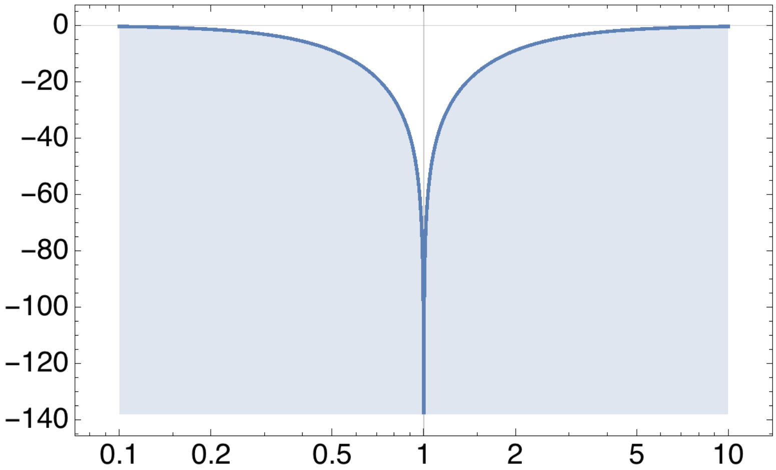
```
H = HLP HLP + HHP HHP // Simplify
```

$$\frac{-1+RC^2\,\omega^2}{(-ⅈ+RC\,\omega)^2}$$

```
HMag = H Conjugate[H] /. RC → 1 // FullSimplify
```

$$\frac{\left(-1+\omega^2\right)^2}{\left(1+\omega^2\right)^2}$$

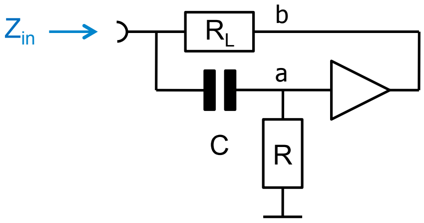
```
LogLinearPlot[dB[HMag], {ω, 0.1, 10}, GridLines → {{1}, {0}}]
```




```
Limit[ArcTan[Im[HLP HLP] / Re[HLP HLP] /. {RC → 1}], ω → 1] // ComplexExpand
```

$\frac{\pi}{2}$

Exercise 1.5 (Gyrator)




```
$Assumptions = R > 0 && RL > 0 && s > 0 && C > 0;

EQin = iin == (vin - va) s C + (vin - vb)/RL /. vb → va;

EQa = (vin - va) s C == va / R;

Eliminate[{EQin, EQa}, va] // Simplify
```

iin (RL + C R RL s) == vin + C RL s vin

```
sol = Solve[%, iin] // First
```

$\left\{iin \to \frac{vin + C\,RL\,s\,vin}{RL\,(1 + C\,R\,s)}\right\}$

```
Zgyrator[s_] = vin/iin /. sol // Simplify
```

$\frac{RL + C\,R\,RL\,s}{1 + C\,RL\,s}$

```
Zapprox[s_] = Numerator[Zgyrator[s]]
```

RL + C R RL s

```
Zapprox[s] /. C → τ/RL // Simplify
```

RL + R s τ

```
Zgyrator[0]
```

RL

```
Limit[Zgyrator[s], s → ∞]
```

R

```
(* i.e. up to corner frequency 1/(C RL), the equivalent schematic is ok *)
```

```
iabs = Sqrt[1/(Zgyrator[i ω] Zgyrator[-i ω])] // FullSimplify
```

$$\sqrt{\frac{1 + C^2\, RL^2\, \omega^2}{RL^2\, \left(1 + C^2\, R^2\, \omega^2\right)}}$$

```
iabs /. {R → RL}
```

$$\sqrt{\frac{1}{RL^2}}$$

```
LogLinearPlot[
 dB[iabs /. {R → 1000, RL → {100, 1000, 10 000}, C → 1 × 10^-9}], {ω, 1 × 10^1, 1 × 10^8}]
```