Some BASICs

Voltage, Current, Components and AC behavior,
Bode Plots, Transfer Functions, Thévenin Equivalent,
High-pass and Low-pass filters,…

Prefixes for Units

- For writing down small or large quantities, exponents can be used: $1.5 \times 10^6\ \Omega$, 3×10^{-9} A
- To simplify, ***prefixes*** in steps of 1000 are used:

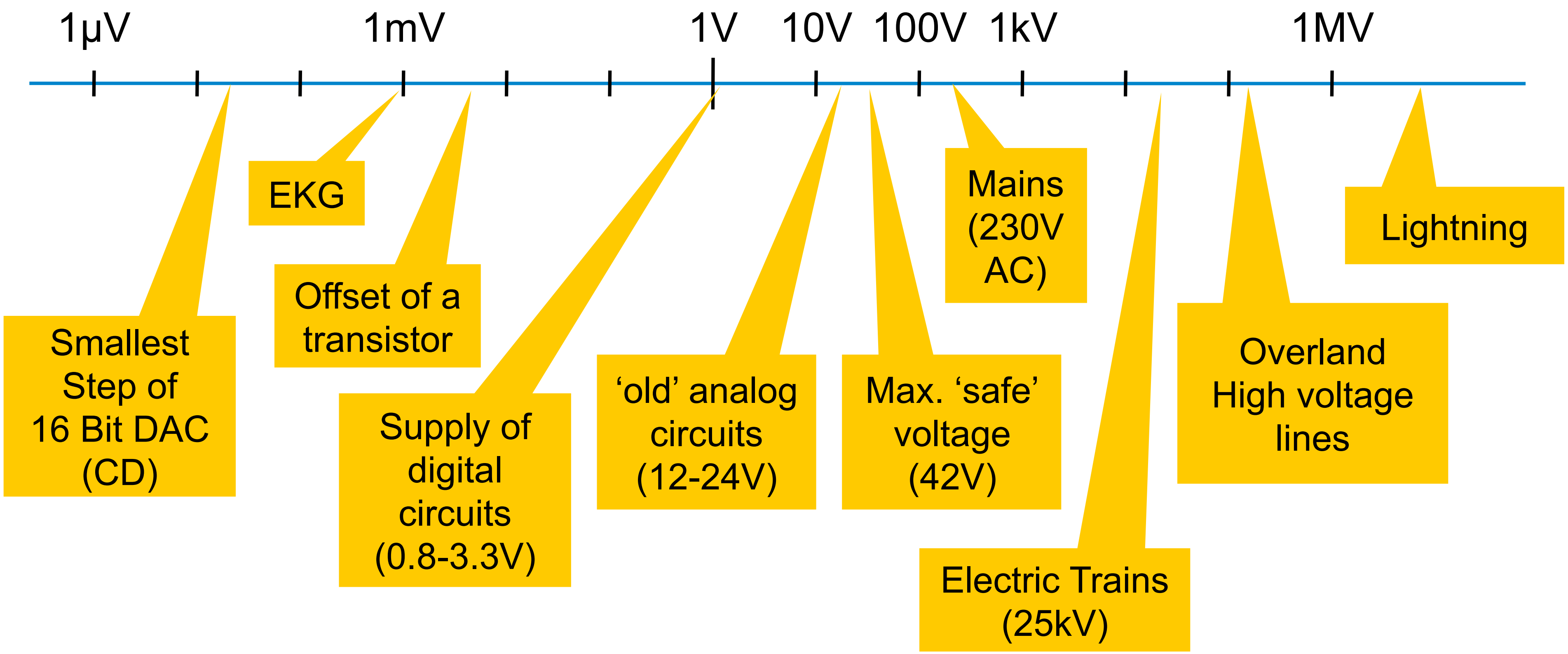
T	Tera	$\times 10^{12}$
G	Giga	$\times 10^{9}$
M	Mega	$\times 10^{6}$
k	Kilo	$\times 10^{3}$
	1	$\times 10^{0}$
m	Milli	$\times 10^{-3}$
µ (or u)	Mikro	$\times 10^{-6}$
n	Nano	$\times 10^{-9}$
p	Piko	$\times 10^{-12}$
f	Femto	$\times 10^{-15}$
a	Atto	$\times 10^{-18}$

- Try to learn: ‘Piko × Kilo = Nano, Milli × Mega = Kilo,…’

Voltage, Current, Kirchhoff's Laws

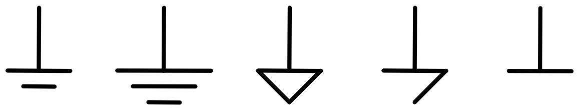
Voltage

- Voltage is the *difference* in electrical potentials, i.e. the energy required to move a unit charge in an electric field
  - This is only well defined in static fields where rot $\vec{E} = \vec{0}$
- Unit: Volt (V)



Ground

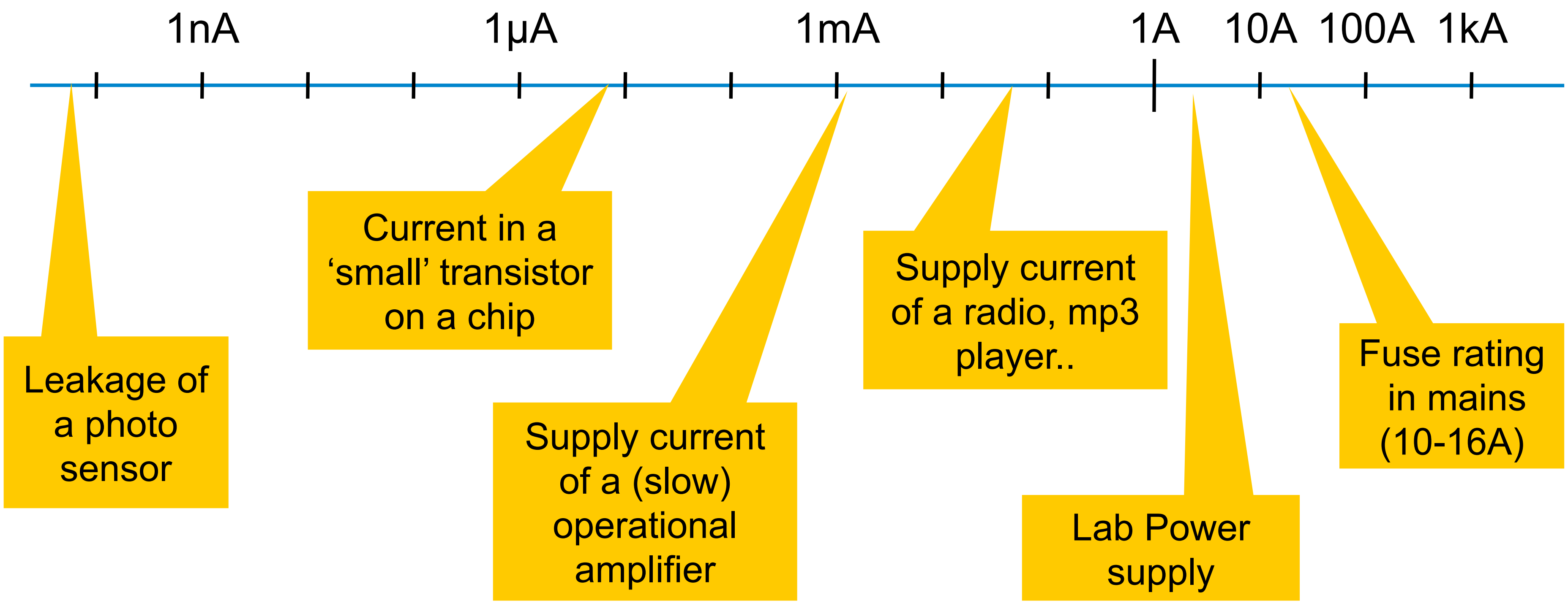
- Voltages are really potential **differences**
- To simplify life, we define a **reference potential** to which voltages are referred. We call it '**ground**'
  - i.e. when we say 'net A has 3V', we mean $V_A - V_{GND} = 3V$
  - Ground is at 0V by definition
- Common ground symbol are:



- (Later we may use several grounds, all at 0V, but separated, for digital and analogue circuit parts)

Current

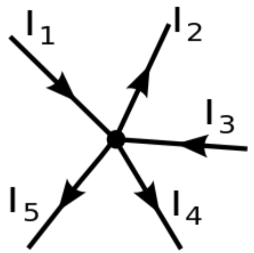
- Electric current is the flow (or change) of electric charge
- $i = dQ / dt$
- Unit: Ampere (A)



Kirchhoff's Laws

1. The sum of currents at any node is zero:

$$\sum_{k=1}^{n} I_k = 0$$

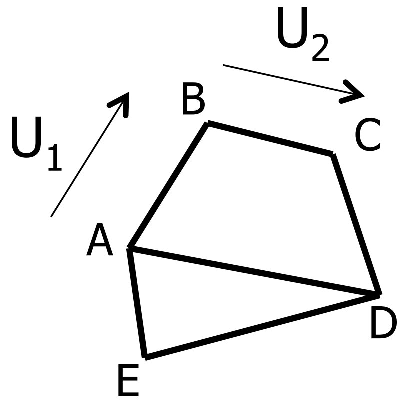


• Follows from charge conservation

2. The sum of voltages in any closed loop is zero:

$$\sum_{k=1}^{n} U_k = 0$$

The sign of the U_k is fixed by a consistent ordering of the nodes in the loop.
Example:
$U_1 = U_B - U_A$, $U_2 = U_C - U_B$, ..
$U_1 + U_2 + U_3 + U_4 = 0$



• Follows from energy conservation

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Resistors & Capacitors

Resistors

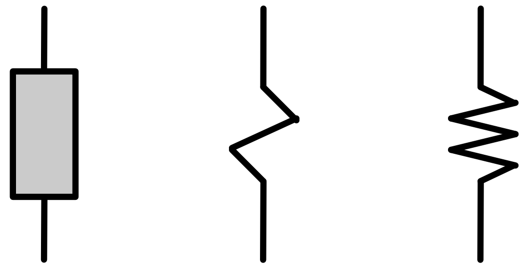
- A resistor is a 2 terminal device
- When voltage is applied, a current flows
- The current is **proportional** to the voltage (Ohm's law):

$I = U \times G$ G is the **conductivity** (Leitwert) in Siemens [S]

or

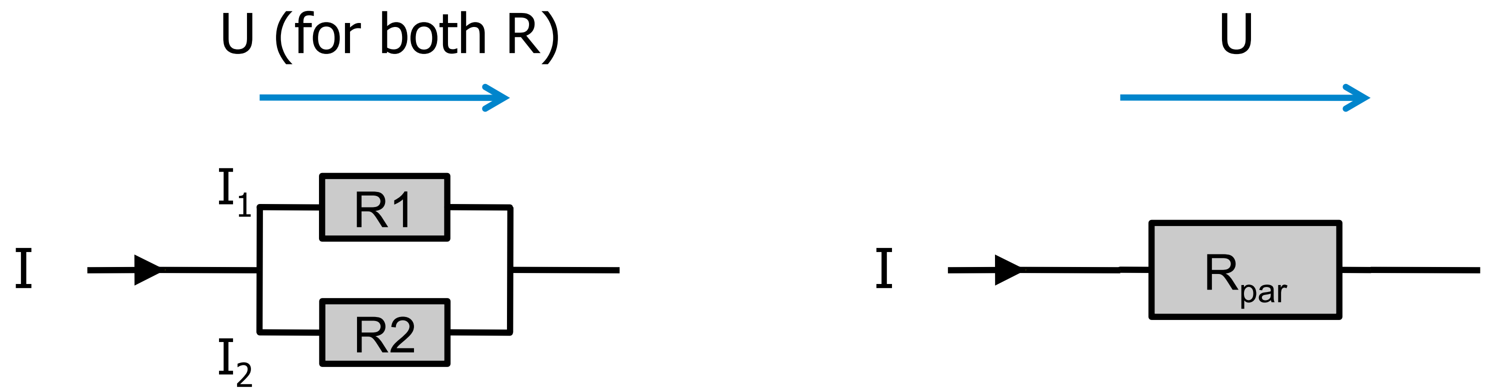
$I = U / R$ R is the **resistivity** (Widerstand) in Ohm [Ω]

- G and R describe the same thing. $G = 1/R$, $R = 1/G$
- Symbols:



- Note: **Ohm's law is not trivial**. Not all materials are 'ohmic'

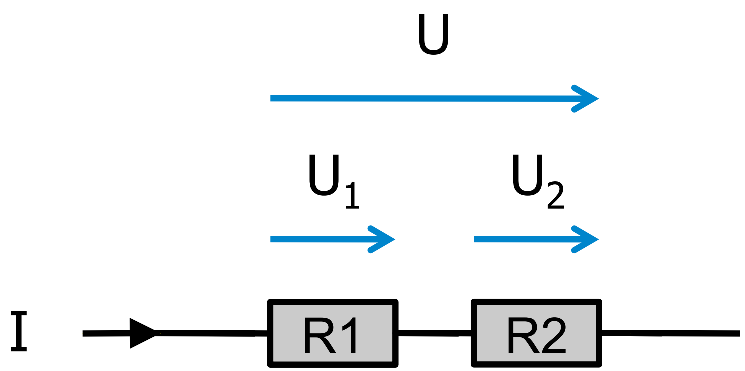
Parallel Connection of Resistors

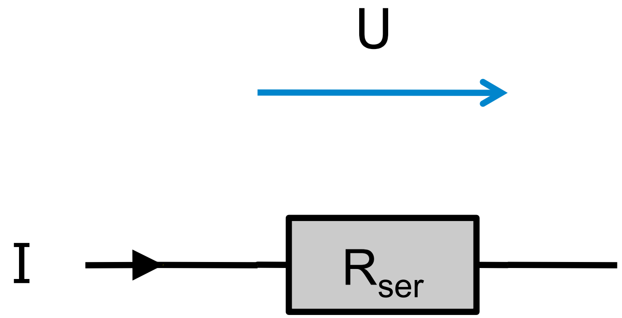


$I = I_1 + I_2 = G_1 \times U + G_2 \times U = (G_1 + G_2) \times U$ $\qquad$ $I = G_{par} \times U$

$$G_{par} = G_1 + G_2 \quad \leftrightarrow \quad 1/R_{par} = 1/R_1 + 1/R_2$$

Series Connection of Resistors



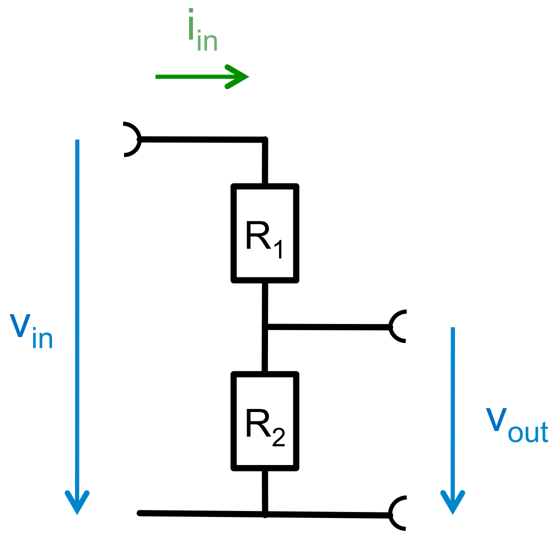


$$U = U_1 + U_2 = I \times R_1 + I \times R_2 = I \times (R_1 + R_2) \qquad U = I \times R_{ser}$$

$$R_{ser} = R_1 + R_2 \quad \leftrightarrow \quad 1/G_{ser} = 1/G_1 + 1/G_2$$

The Voltage Divider (***without*** load current!)

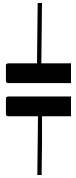
- A very common topology is the voltage divider:

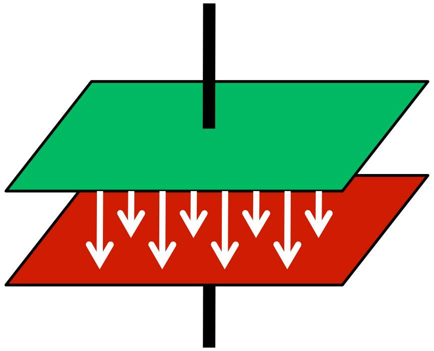


  - The input current $i_{in} = v_{in} / (R_1+R_2)$
  - This current flows through R_1 and R_2, i.e. $i_{in} = i_{R1} = i_{R2}$
  - On R_2, it develops a voltage $v_{out} = i_{R2} R_2 = v_{in} R_2 / (R_1+R_2)$

- Overall: $v_{out} / v_{in} = R_2 / (R_1+R_2)$
  - Remember: The 'gain' is the *value of the resistor where we measure* divided by the *total resistance*

Capacitors





- A capacitor can store electrical charge
- Prototype: parallel plate capacitor
  - **Charge Q** on plates generates field (through Gauss' law)
  - Field between plates gives a **voltage V**
- Q = C × V: capacitance is factor between charge and voltage
  - A **large** capacitor can store a **lot** of charge at **low** voltage

- The voltage on a capacitor is given by the current integral:

$$\boxed{V = \frac{Q}{C} = \frac{1}{C}\int I(t)dt \quad \Leftrightarrow \quad I(t) = C\frac{dV}{dt}}$$

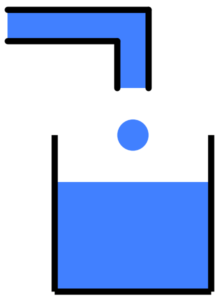
- The stored energy is:

$$dE(Q) = V(Q)dQ \Rightarrow E = \int_0^Q V(Q')dQ' = \int_0^Q \frac{Q'}{C}dQ' = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

Water Analogy

- A well working analogy using water is:

Current	⇔	Water flow (m^3 / second or so)
Voltage	⇔	Water level (m)
Capacitance	⇔	Area of a container (m^2)
$V = T\,I / C$	⇔	Level = T Flow / Size

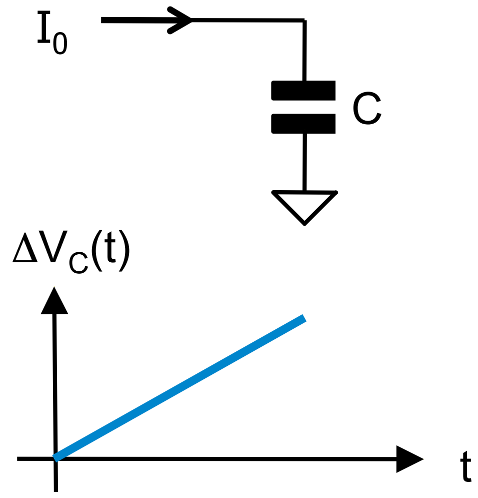


- Water flowing in the container leads to rising water level.
  - Higher flow → faster increase of voltage level
  - Larger container → slower increase
- With this model, we can also visualize nonlinear caps:

Charging a Capacitor (important!)

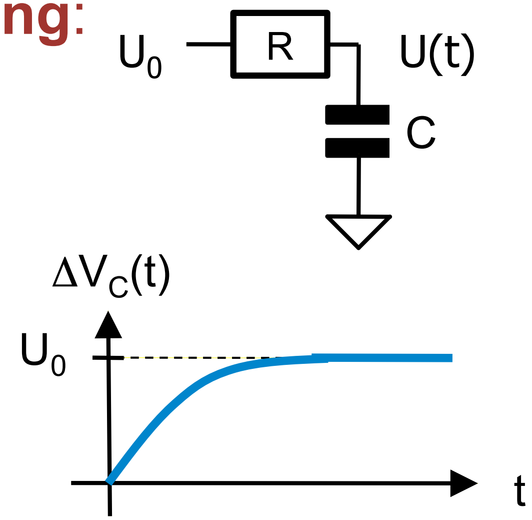
- At constant current I: **linear ramp**:

$$
\begin{aligned}
I(t) &= I_0 = \text{const} \\
\Delta Q(t) &= \int_0^t I(t')\,dt' = \int_0^t I_0\,dt = I_0 \times t \\
\Delta U(t) &= \frac{\Delta Q(t)}{C} = \frac{I_0}{C} \times t
\end{aligned}
$$



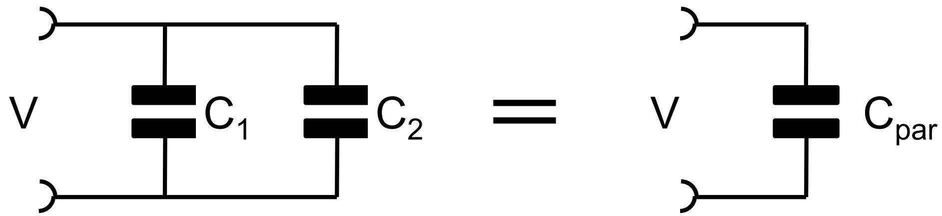
- Through resistor R: **exponential settling**:

$$
\begin{aligned}
I(t) &= \frac{U_0 - U(t)}{R} \\
\frac{dU(t)}{dt} &= \frac{I(t)}{C} = \frac{U_0 - U(t)}{RC} \\
\text{Solution}: \; U(t) &= U_0 - U_0\, e^{-\frac{t}{RC}}
\end{aligned}
$$

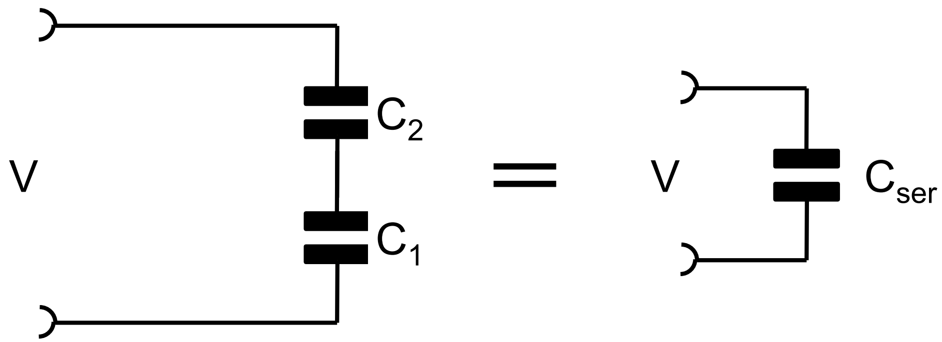


Parallel and Series Connection of Capacitors

- For derivation, see exercise…



$$C_{par} = C_1 + C_2$$

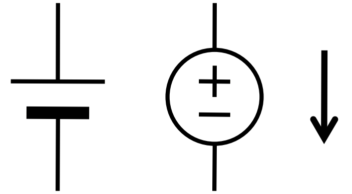


$$\frac{1}{C_{ser}} = \frac{1}{C_1} + \frac{1}{C_2}$$

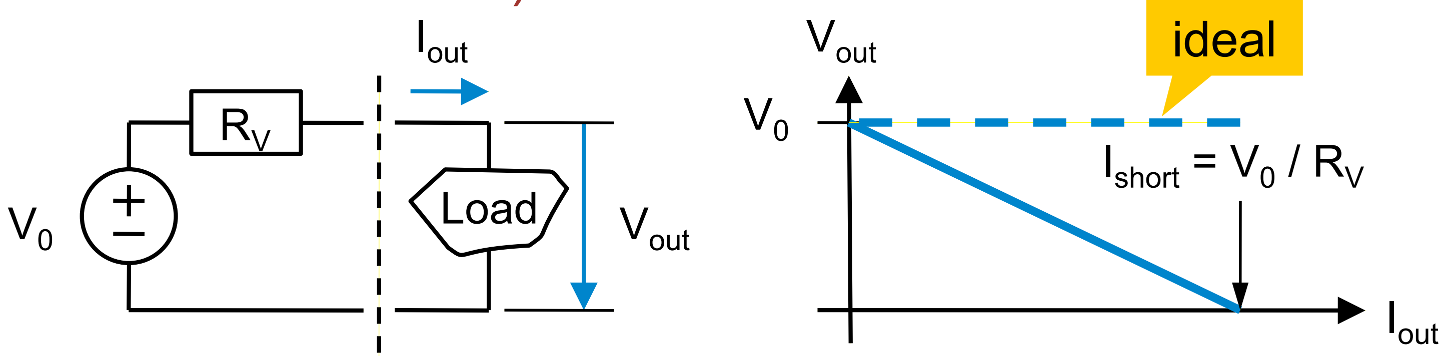
VOLTAGE & CURRENT SOURCES

Voltage Sources

- A voltage source has 2 terminals:



- An **ideal** voltage source maintains the voltage for **any** output current ('1000 A')
- The voltage of a **real** source drops with **load current**.
- This is modeled by a **series** resistor (internal resistor, source resistor):



- The **open voltage** is V_0 (I_{out}=0 → voltage drop over R_V is 0)
- The **short circuit current** is $I_{short} = V_0 / R_V$
- Note: '**Good**' voltage sources have **low** $R_V \rightarrow 0$

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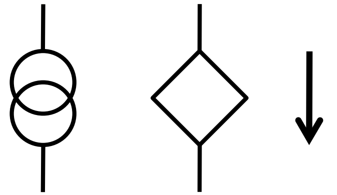
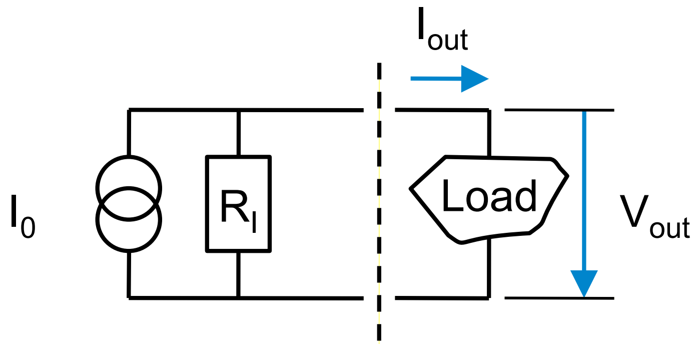
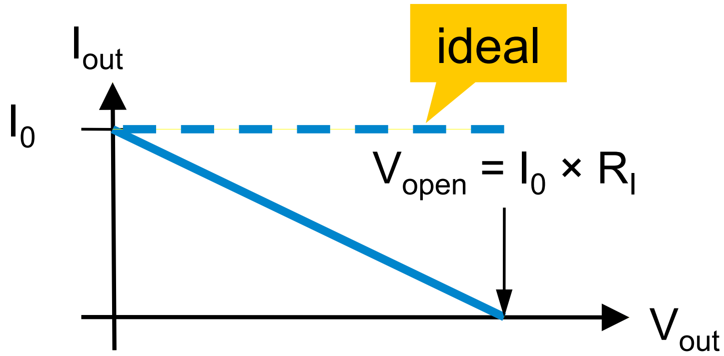
Current Sources

- A current source has 2 terminals:



- An **ideal** current source maintains the current for **any** output voltage
- The current of a **real** source drops with **load voltage**.
- This is modeled by a **parallel** resistor (internal resistor, source resistor):

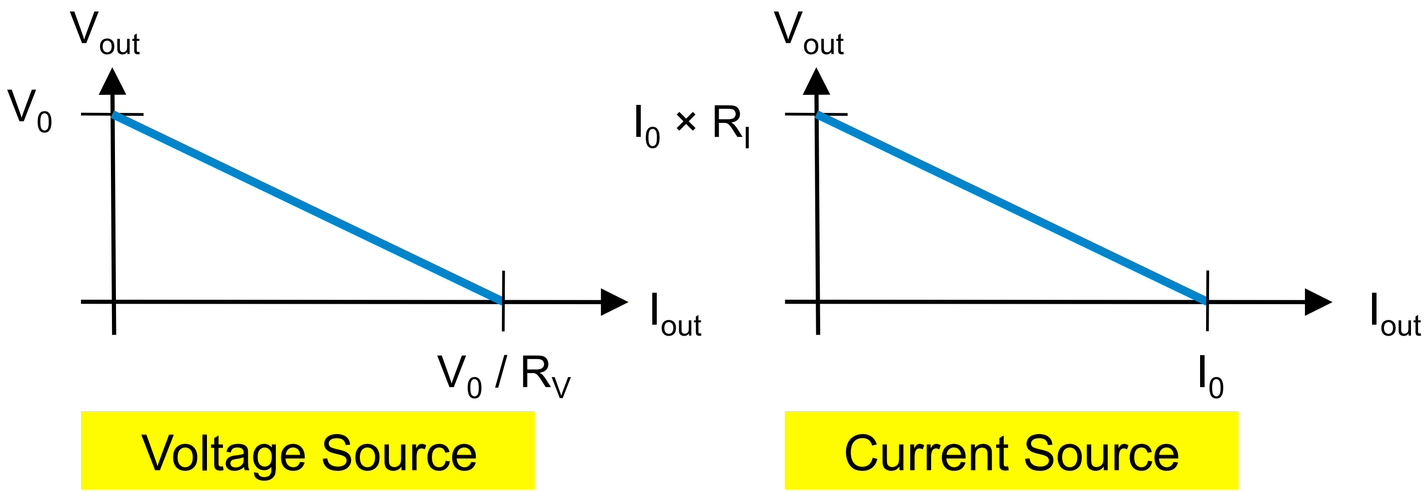




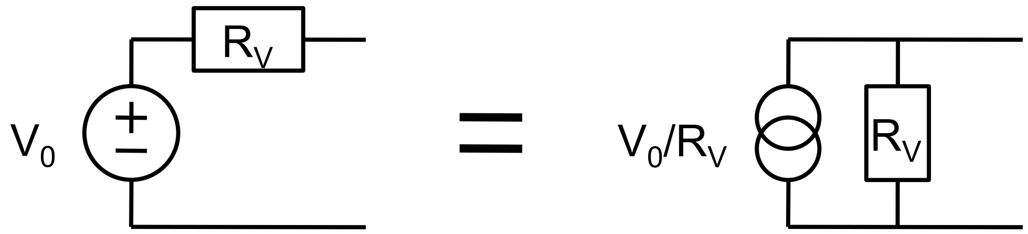
- The **short circuit current** is I_0 (no voltage at R_I→no current)
- At a voltage of $I_0 \times R_I$ no more current flows (all flows in R_I)
- Note: ‘**Good**’ current sources have **high** $R_I \rightarrow \infty$

Equivalence of U- and I-Source

- Flip the diagram of the I-source and compare:



- Same shape! Therefore:
- For voltage source with V_0 and R_V, a current source with $I_0 = V_0 / R_V$ and $R_I = V_0 / I_0 = R_V$ behaves the same!

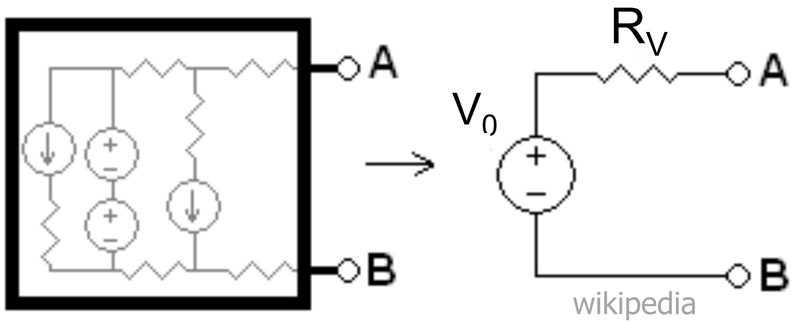


Thévenin's Theorem

Any combination of U-sources, I-sources and resistors behaves like a (real) voltage source with an internal resistor

- This is fairly obvious from the previous page and the linearity of the resistor properties
- Obviously, a current source with internal resistor can also be used

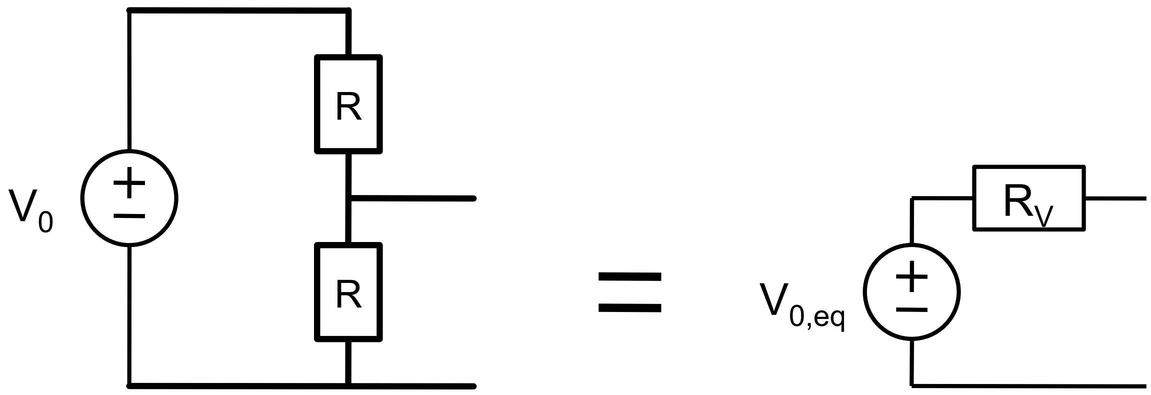
- Example:



- To find V_0: calculate the open voltage
- To find R_V: find the short circuit current. Then $R_V = V_0 / I_{short}$

Thévenin Equivalent of a Voltage Divider

- Consider a voltage divider with two equal resistors:

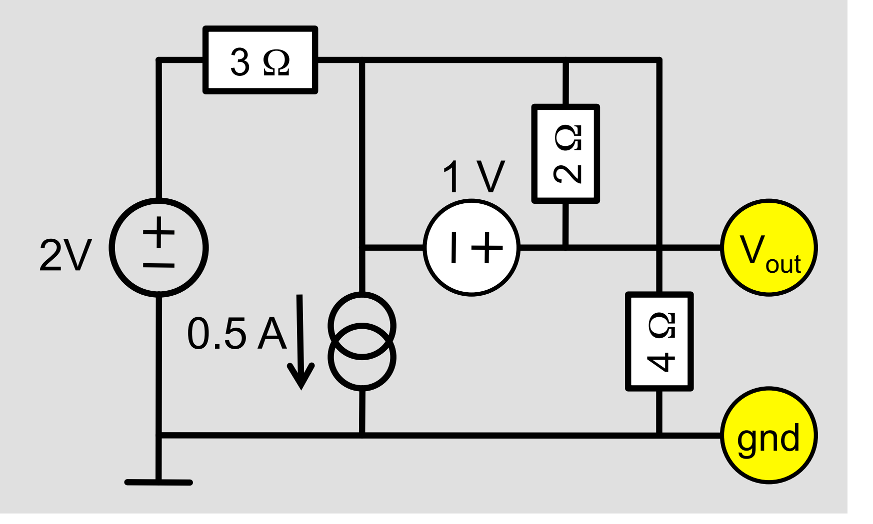


- $V_{0,eq} = V_0 / 2$ $\quad$ $(I = V_0 / (2R),\ V_{0,eq} = R \times I)$
- $I_{short} = V_0 / R \rightarrow R_V = V_{0,eq} / I_{short} = R / 2$

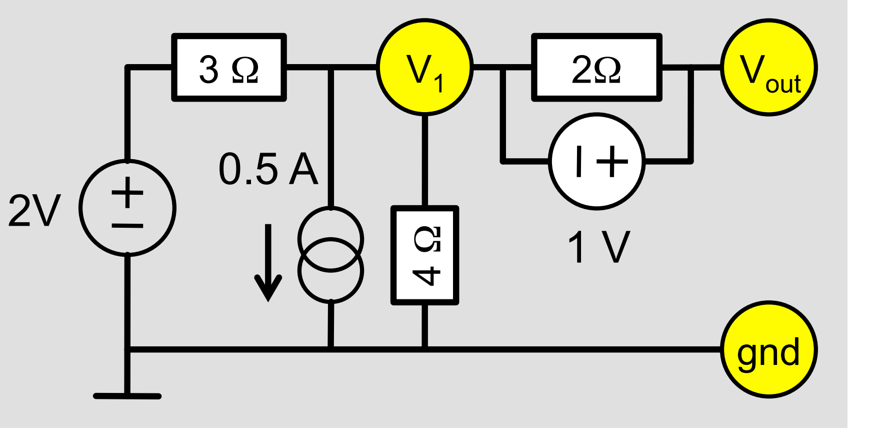
- In the general, case R_V is the parallel connection of R_1 and R_2. Remember that!

A More Complicated Example

- What is the Thévenin equivalent of this circuit ?



- Before we start
  - Label nodes
  - Re-draw schematics for better understanding:



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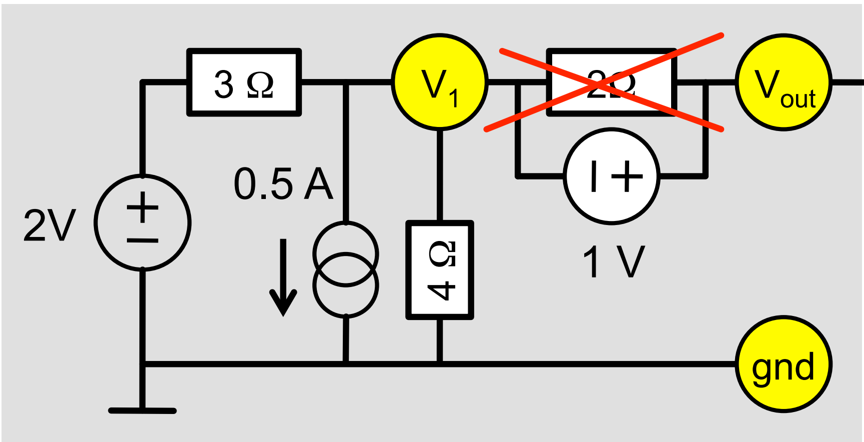
A More Complicated Example

- Open circuit:

$(2V\text{-}V_1) / 3\Omega \longrightarrow$



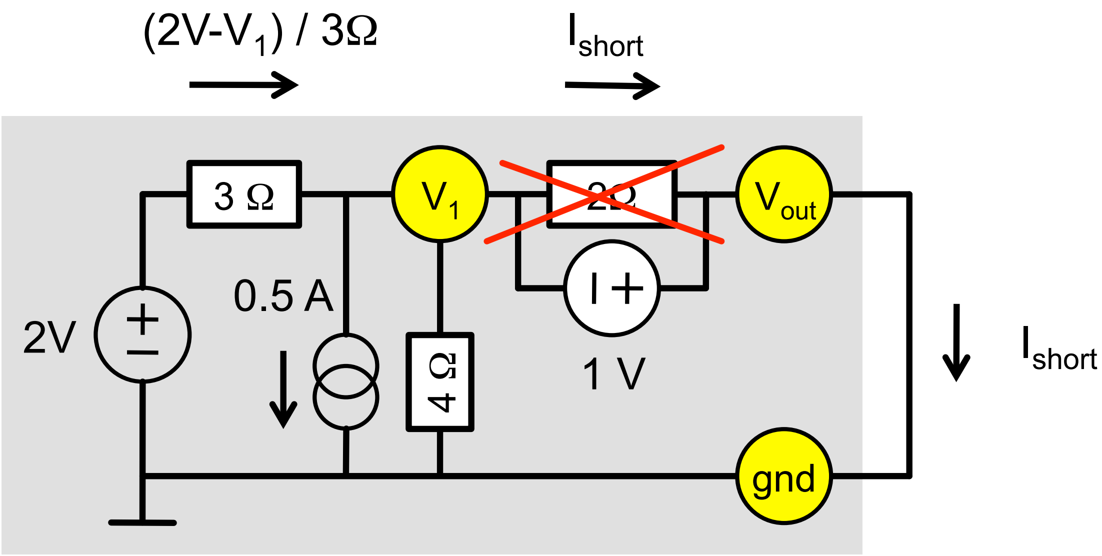
$\longrightarrow I_{out} = 0$

- Current sum at note V_1:
  - $(2V\text{-}V_1) / 3\ \Omega = 0.5\ A + V_1 / 4\ \Omega \rightarrow V_1 = 0.285..\ V$

- $V_{out}\ (I_{out}{=}0) = V_1 + 1V = 1.285..\ V = V_{0,eq}$

A More Complicated Example

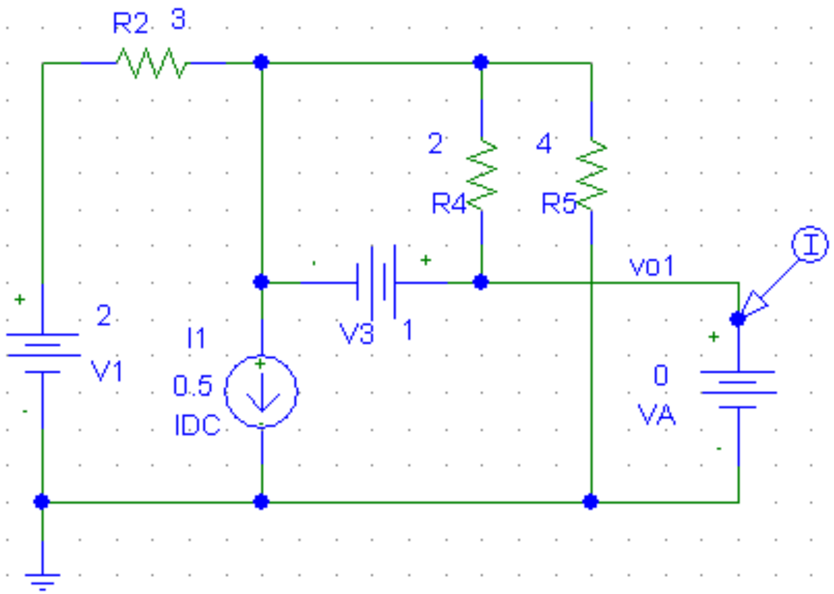
- Short circuit:

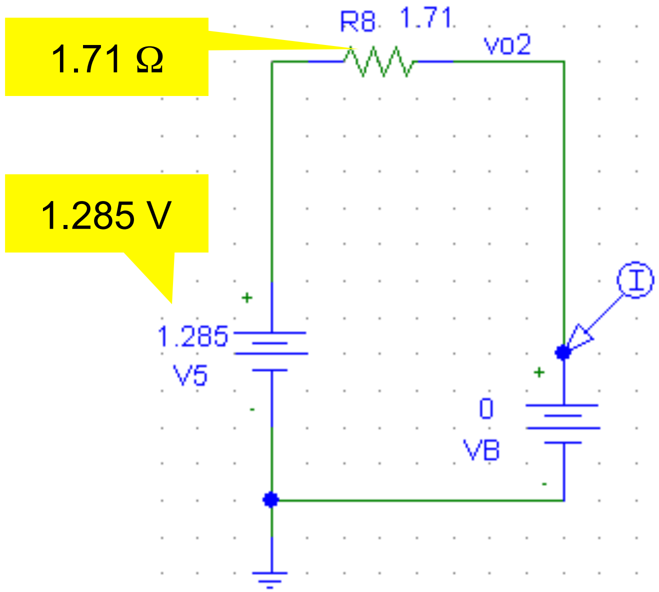


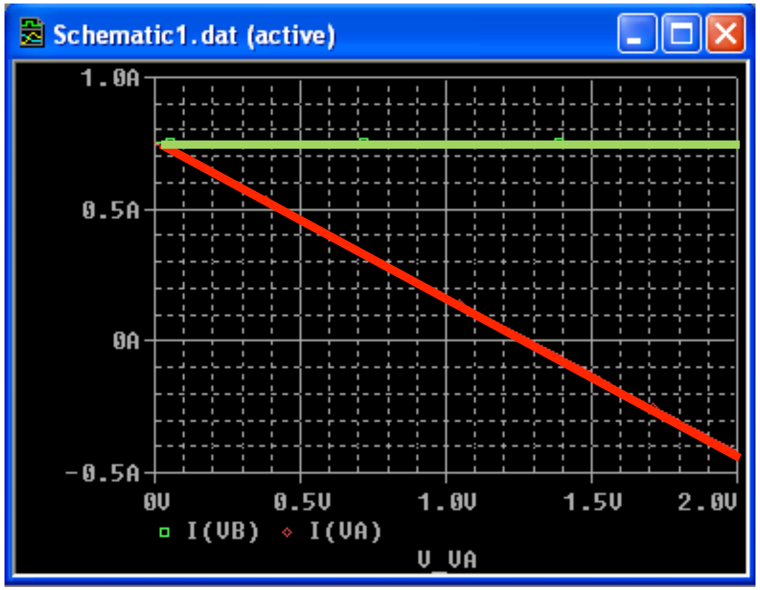
- Again current sum at note V_1:
  - $(2V-V_1) / 3\ \Omega = 0.5\ A + V_1 / 4\ \Omega + I_{short} \rightarrow I_{short} = 0.75\ A$

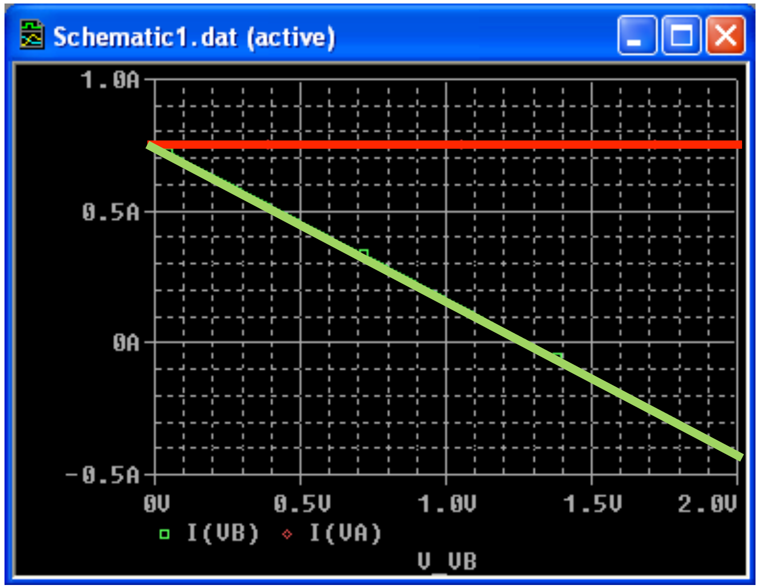
- $R_V = V_{0,eq} / I_{short} = 1.285..\ V / 0.75\ A = 1.71\ \Omega$

A More Complicated Example - Simulation









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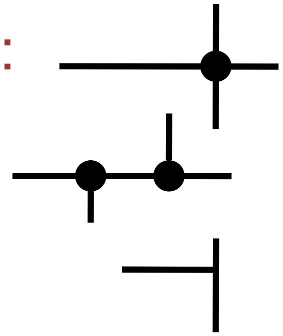
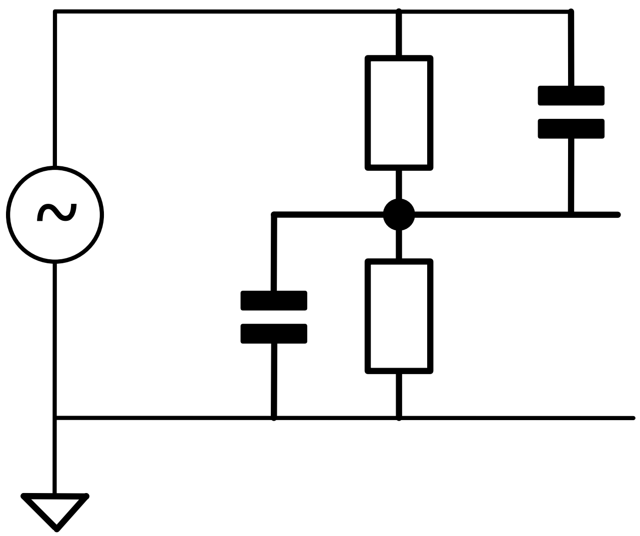
DRAWING SCHEMATICS

Drawing Schematics: Some Rules

- **Positive** voltages are at the **top**, negative at the bottom
- **Input** signals are at the **left**, **outputs** at the **right**
- Connected **crossings** are marked with a ● :
  - should be avoided
- T-connections do not need a ● :
  - but they can have one…





Example for a ‘nice’ schematic

- A useless circuit…