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Exercise 1:
Thévenin Equivalent & RC-Circuits

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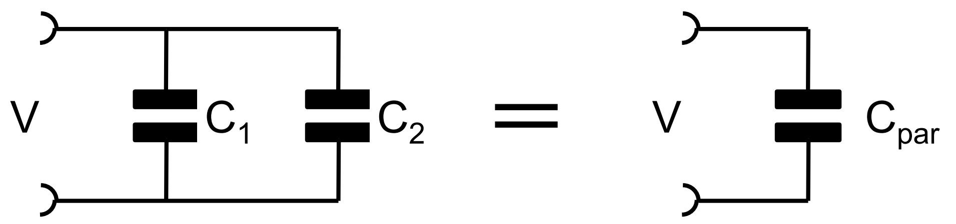
Recommendations

- I strongly recommend to use a mathematical program (Mathematica, Maple, SageMath,..) to solve the exercises

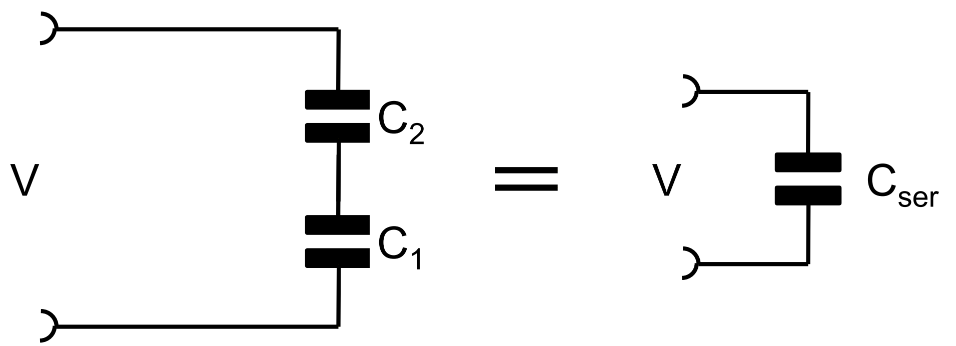
- For transfer functions, inspect each result:
  - What happens for $\omega \rightarrow 0,\infty$?
  - What happens if component values go to 0 or ∞?

Exercise 1.0

- Derive the expressions for the series and parallel connection of capacitors using
  - Charge conservation
  - Complex impedance & Kirchhoff's law

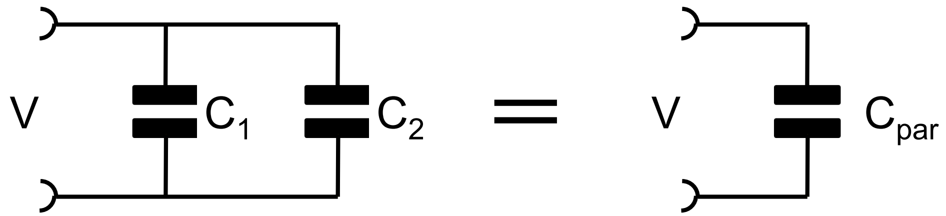


C_{par} = ???



C_{ser} = ???

Solution 1.0



C_{par} = ???

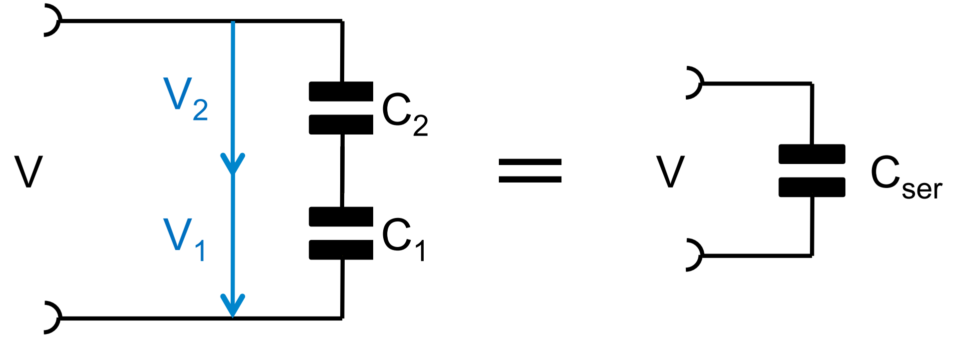
1. Charge conservation:

$V \times C_1 + V \times C_2 = Q_1 + Q_2 = Q_{par} = V \times C_{par} \quad \rightarrow \quad C_1 + C_2 = C_{par}$

2. Kirchhoff & complex impedance:

$V\, sC_1 + V\, sC_2 = i_1 + i_2 = i_{par} = V\, sC_{par} \quad \rightarrow \quad C_1 + C_2 = C_{par}$

Solution 1.0



C_{ser} = ???

1. Charge conservation:

Note: no charge can 'escape' the middle node, so that $Q_1=Q_2=Q_{ser}$

$V = V_1 + V_2 = Q_1/C_1 + Q_2/C_2 = Q/C_1 + Q/C_2 = Q/C_{ser}$

$\rightarrow \quad 1/C_1 + 1/C_2 = 1/C_{ser}$

2. Kirchhoff & complex impedance:

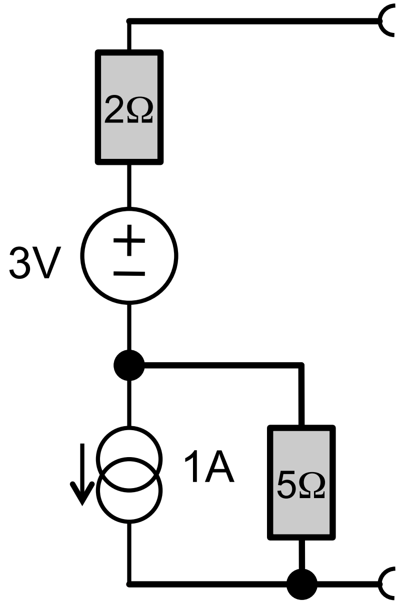
$V_1\, sC_1 = V_2\, sC_2$ and $V_1 + V_2 = V \quad \rightarrow \quad V_1 = V\, C_2 / (C_1+C_2)$

$\rightarrow \quad i_1 = V_1\, sC_1 = V\, s\, C_1C_2 / (C_1+C_2)$

$\rightarrow \quad C_{ser} = i / (Vs) = i_1 / (Vs) = C_1C_2 / (C_1+C_2)$

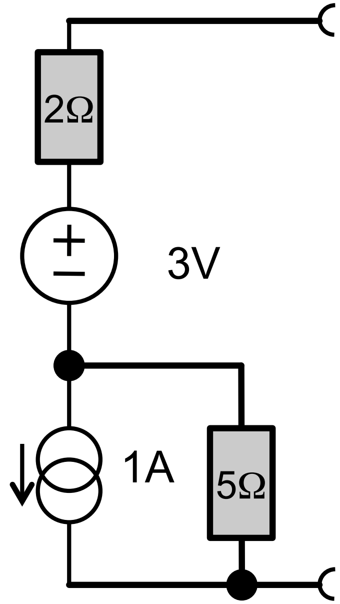
Exercise 1.1

- Derive the Thévenin Equivalent for the following circuit:



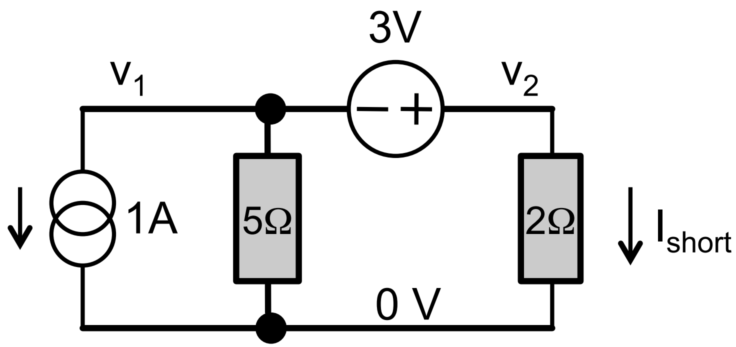
- Try two different methods:
  - Use the Open/Short method with Kirchhoff's rules
  - Convert the I-source part to a voltage source first…

Solution 1.1 – Kirchhoff



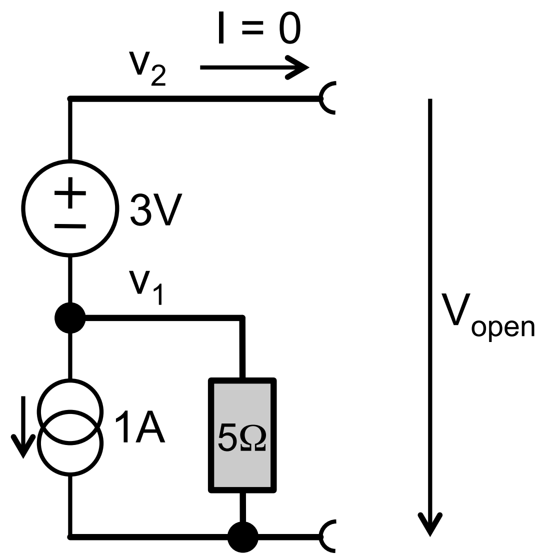
1. Short circuit current:

- EQ1: $1\ A + v_1 / 5\Omega + v_2 / 2\Omega = 0$
- EQ2: $v_2 = v_1 + 3V$
- $\rightarrow v_2 = -4/7\ V$
- $\rightarrow I_{short} = -2/7\ A$



2. Open circuit voltage:

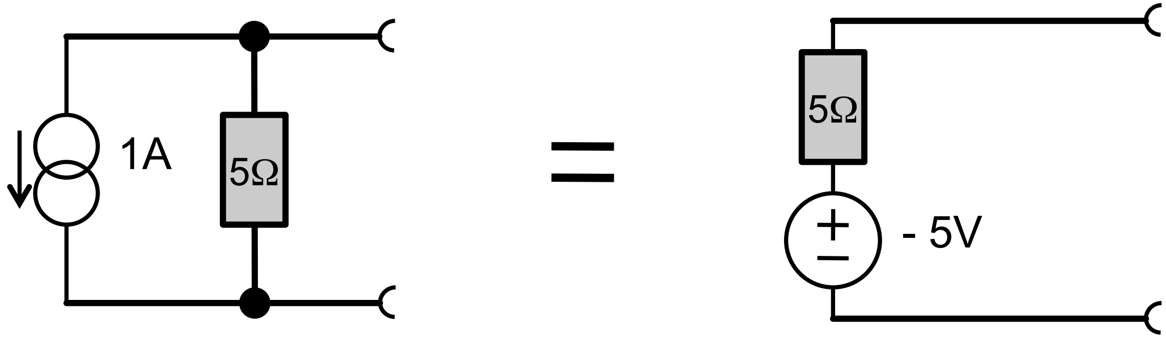
- EQ1: $1\ A + v_1 / 5\Omega = 0$
- EQ2: $v_2 = v_1 + 3V$
- $\rightarrow v_1 = -5\ V$
- $\rightarrow v_2 = V_{open} = -2\ V$



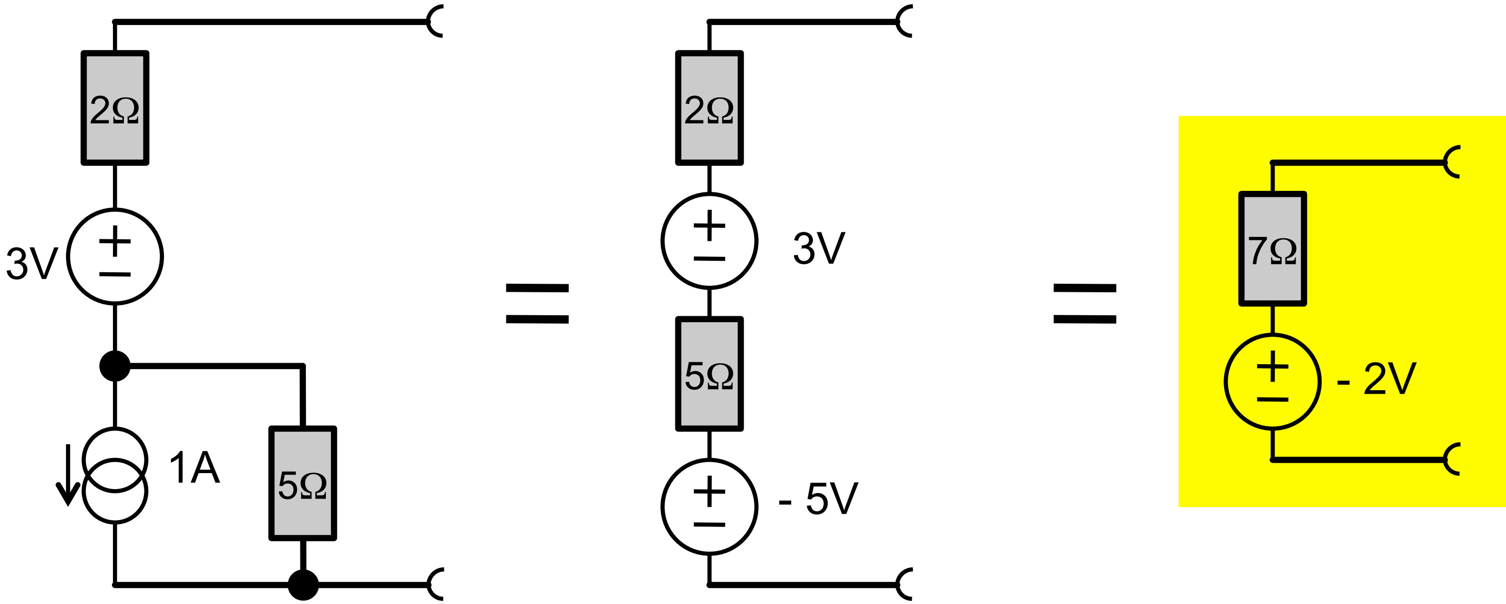
- Source: $V_0 = V_{open} = -2\ V$, $R_V = V_0 / I_{short} = 7\ \Omega$

Solution 1.1 – Thévenin Transformations

1. Convert the current source to a voltage source:

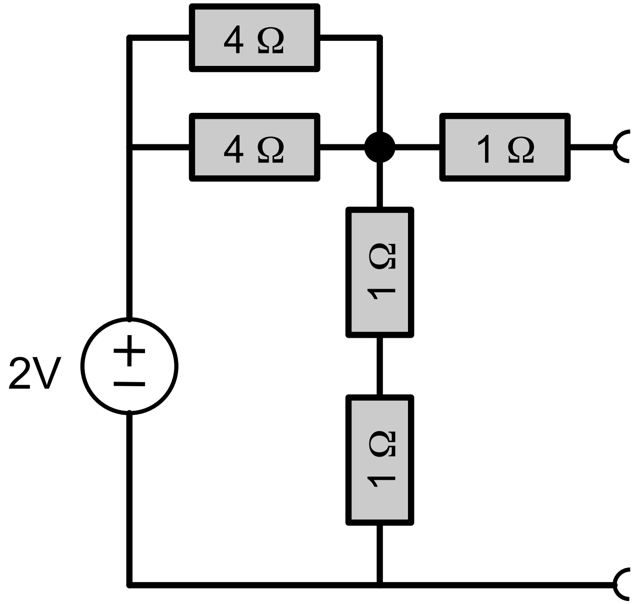


2. Use this in the circuit:



Exercise 1.2

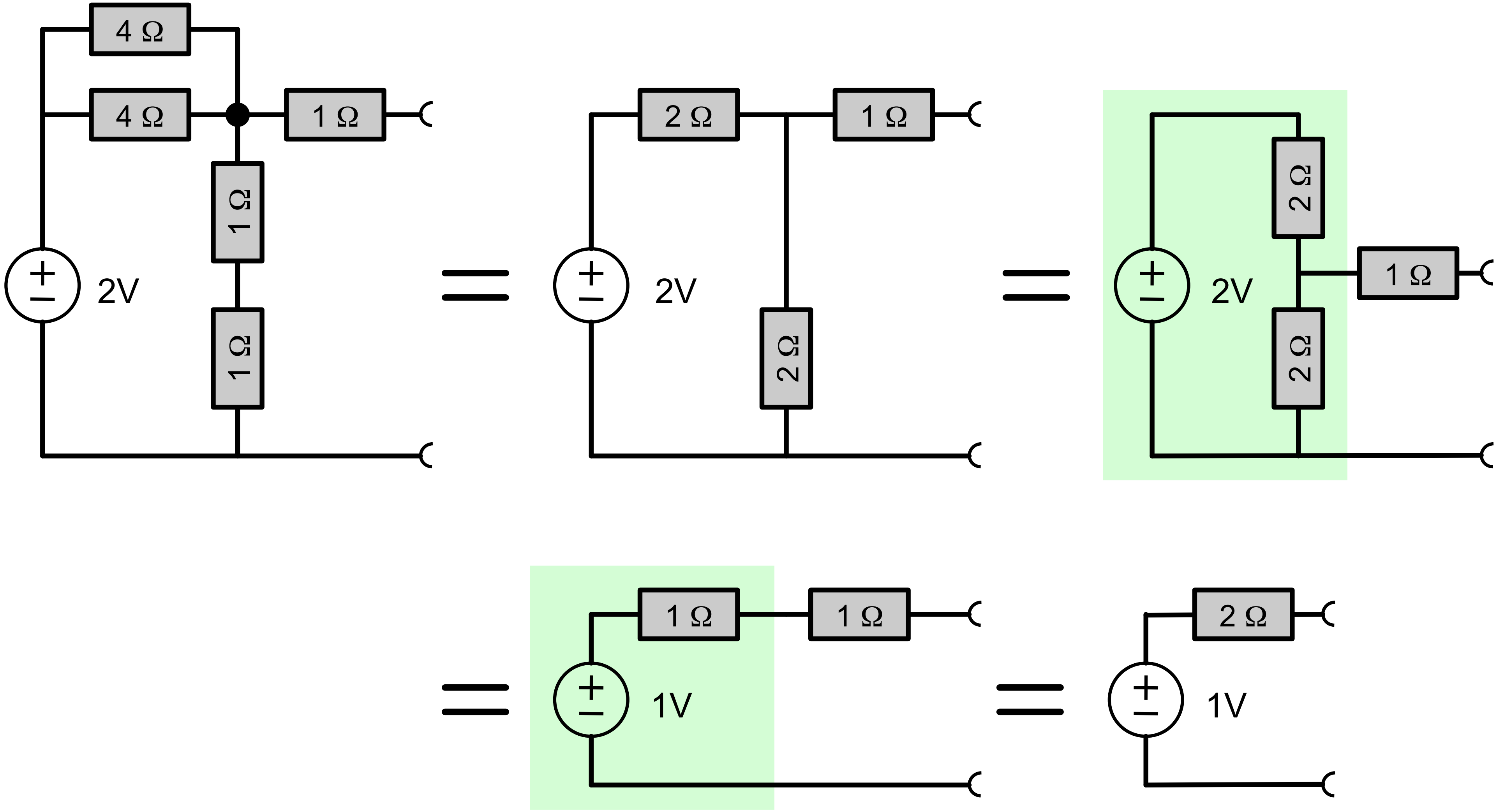
- What is the Thévenin Equivalent of the following circuit?



- Use two methods to find the result:
  - parallel / series connection of resistors and your knowledge about the voltage divider
  - short/open method

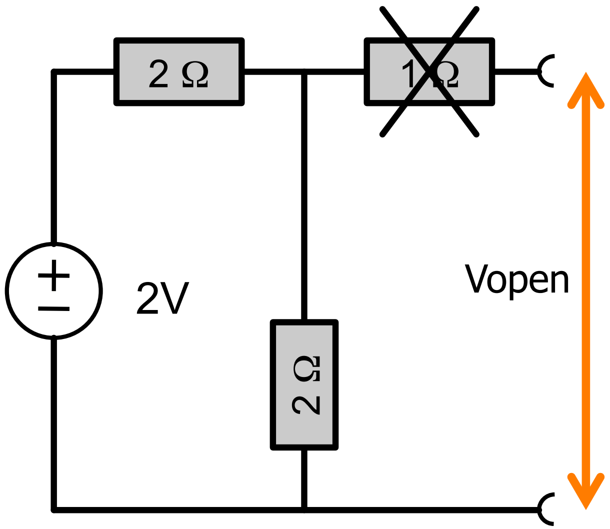
Solution 1.2

- Parallel-Series Connection, Voltage Divider:

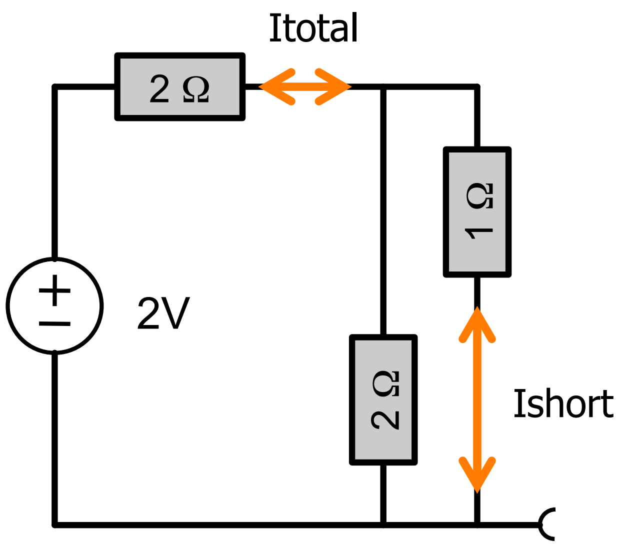


Solution 1.2

- Open: Vopen = 1V



Short:



Rtotal = 2Ω + 2/3Ω = 8/3 Ω

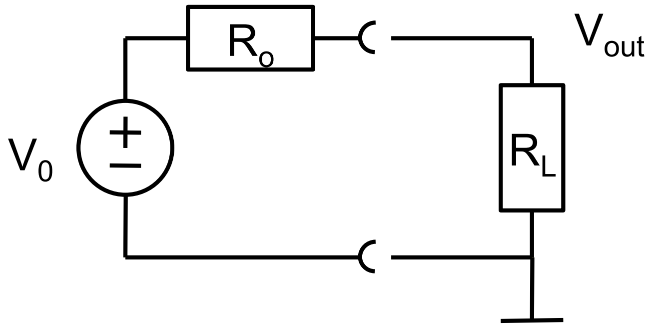
Itotal = 2V / Rtotal = 3/4 A

Ishort = 2/3 Itotal = 1/2 A

Zin = Vopen / Ishort
= 1V / ½ A
= 2 Ω

Exercise 1.3

- A voltage source with voltage V_0 and output resistance R_0 is loaded by a resistor R_L:



- What is the output voltage V_{out} ?
- Which current flows in R_L ?
- What power is dissipated in R_L ?
  - Check that noting is dissipated for R_L=0 and $R_L \rightarrow \infty$
- For which value of R_L is the dissipation maximized?
  - What is the dissipation?

Solution 1.3

In[29]:= Vout = V0 $\frac{RL}{R0 + RL}$;

In[30]:= Iout = $\frac{Vout}{RL}$

Out[30]= $\frac{V0}{R0 + RL}$

In[31]:= Pout = Vout Iout

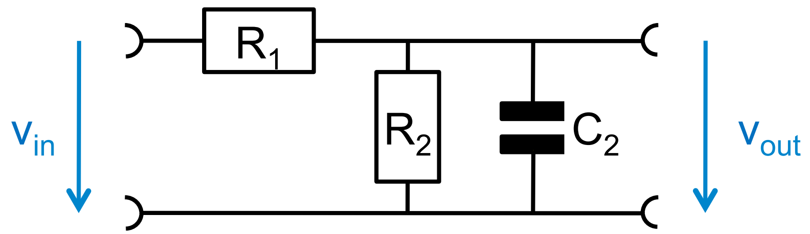
Out[31]= $\frac{RL\ V0^2}{(R0 + RL)^2}$

```
In[38]:= Table[Limit[Pout, RL → x], {x, {0, ∞}}]

Out[38]= {0, 0}

In[39]:= Solve[D[Pout, RL] == 0, RL] // First

Out[39]= {RL → R0}

In[40]:= Pout /. %
```

Out[40]= $\frac{V0^2}{4\ R0}$

Exercise 1.4

- Derive the Transfer Function of this circuit:



- Use 3 different approaches:
  - Treat the circuit directly (using Kirchhoff's rule)
  - Consider it as a voltage divider of two Impedances. Use R_1 for Z_1 and the parallel connection of R_2 and C_2 for Z_2
  - Replace the (resistive) voltage divider by its Thévenin equivalent and then add the capacitor

- Make a Bode Plot
  - Describe the difference to the normal Low Pass Filter

Solution 1.4

Direct Treatment:

$$\texttt{EQ} = \frac{\texttt{Vin} - \texttt{Vout}}{\texttt{R1}} == \texttt{Vout s C2} + \frac{\texttt{Vout}}{\texttt{R2}};$$

```
Solve[EQ, Vout] // First
```

$$\left\{\texttt{Vout} \to \frac{\texttt{R2 Vin}}{\texttt{R1 + R2 + C2 R1 R2 s}}\right\}$$

$$\texttt{Hdirect} = \frac{\texttt{Vout}}{\texttt{Vin}} \ /. \ \%$$

$$\frac{\texttt{R2}}{\texttt{R1 + R2 + C2 R1 R2 s}}$$

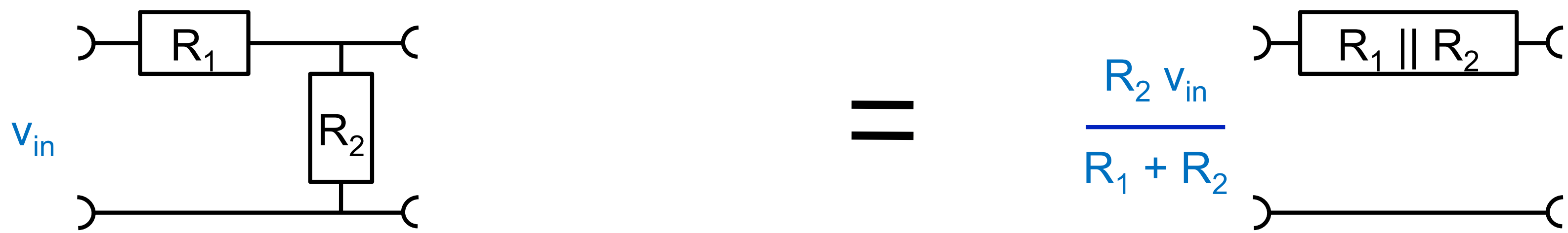
Voltage Divider:

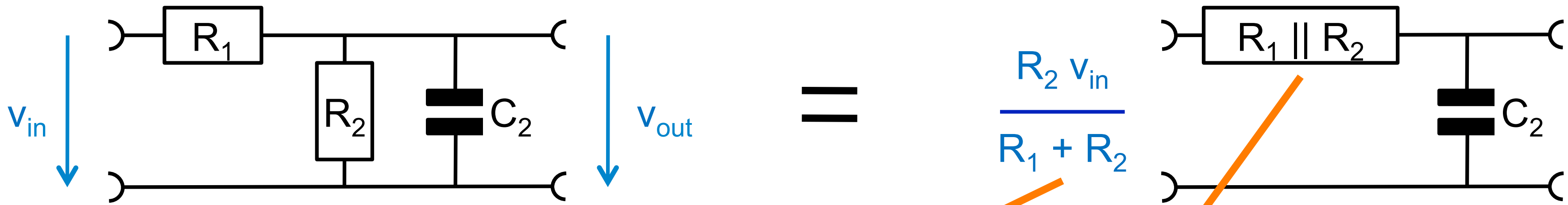
$$\texttt{Hdiv} = \frac{\texttt{Z2}}{\texttt{Z1 + Z2}} \ /. \ \left\{\texttt{Z1} \to \texttt{R1},\ \texttt{Z2} \to \left(\frac{1}{\texttt{R2}} + \texttt{s C2}\right)^{-1}\right\} \ \texttt{// Simplify}$$

$$\frac{\texttt{R2}}{\texttt{R1 + R2 + C2 R1 R2 s}}$$

```
Hdirect == Hdiv
True
```

Solution 1.4





$$\text{Hthenevin} = \frac{g}{1 + s\,RR\,C2} \;/.\; \left\{ g \to \frac{R2}{R1 + R2},\; RR \to \left(\frac{1}{R1} + \frac{1}{R2}\right)^{-1} \right\} \;//\; \text{Simplify}$$

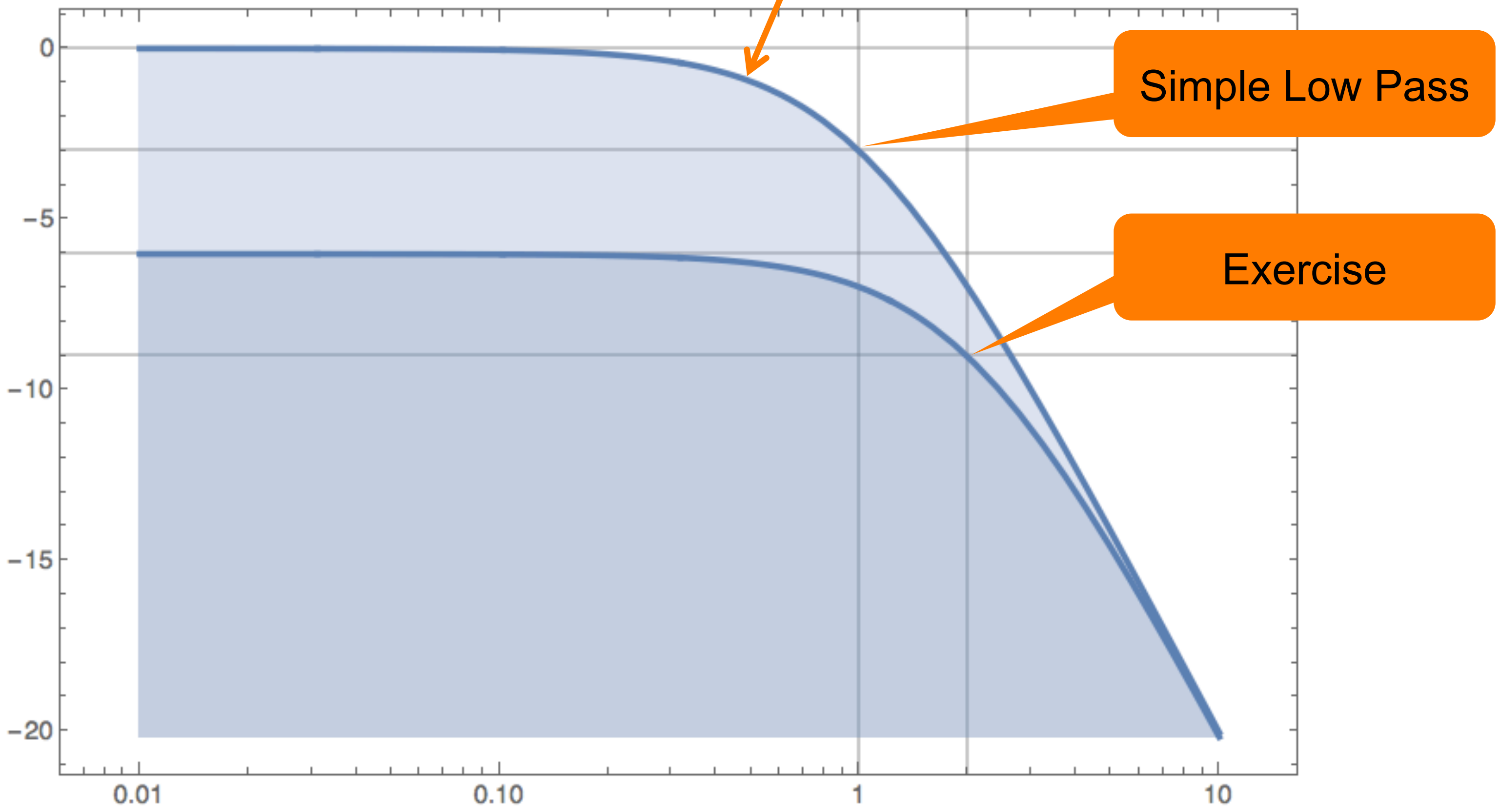
$$\frac{R2}{R1 + R2 + C2\,R1\,R2\,s}$$

Simple Low Pass

Solution 1.4

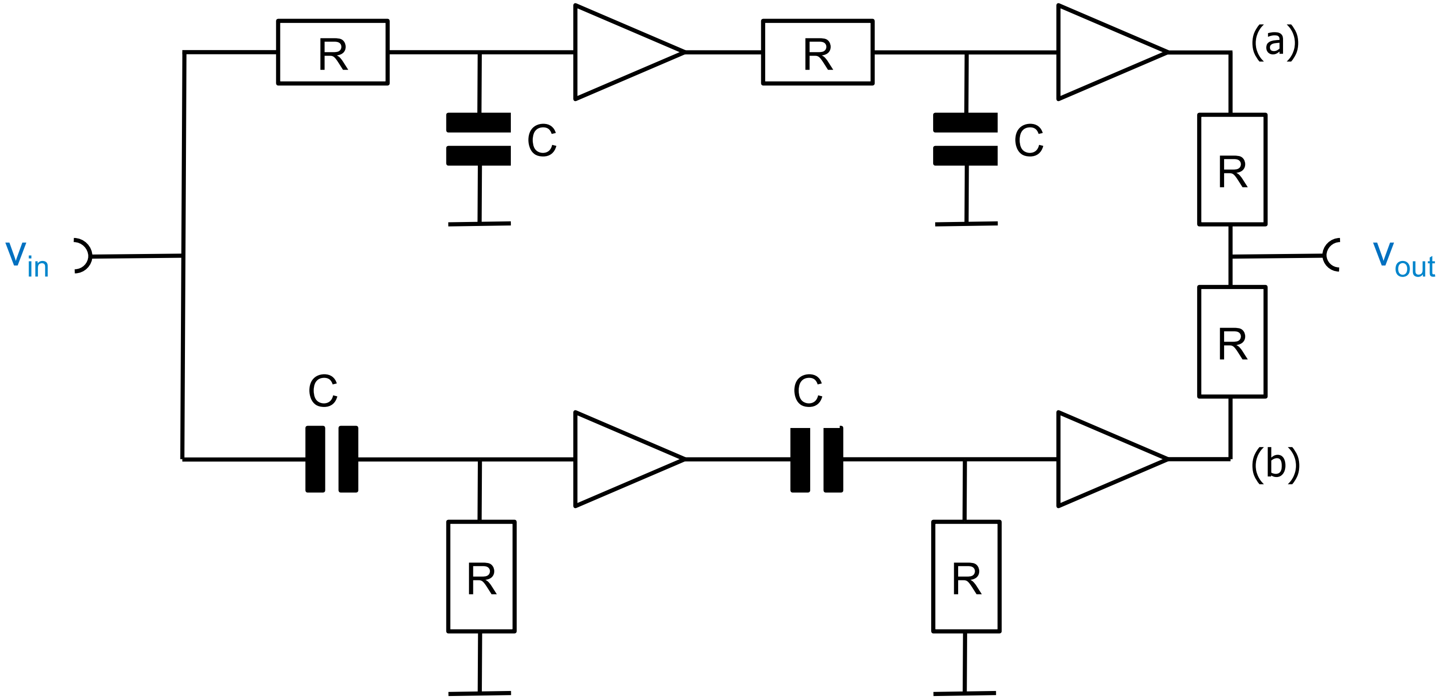
- Compared to the 'simple' Low-Pass:
  - The signal is attenuated by $R_1/(R_1+R_2)$
  - The time constant is lowered (i.e. the corner frequency is raised)
- Plot for $R_1 = R_2 = C_2 = 1$:

$$\mathrm{HLP} = \sqrt{\frac{1}{1+i\,\omega\,RC}\,\mathrm{Conjugate}\left[\frac{1}{1+i\,\omega\,RC}\right]}\ /.\ \{RC \to 1\}$$



Exercise 1.5: Notch Filter

- Consider the following circuit made of cascaded High- and Low Pass stages:
  - The resistors at the output just add the signals at (a) and (b)



- What is the output signal at the corner frequency?
  - Explain this by comparing amplitudes ***and phases*** at (a) and (b)

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Solution 1.5

```
$Assumptions = ω > 0 && RC > 0; HLP = 1/(1 + i ω RC); HHP = (i ω RC)/(1 + i ω RC);

vout = Vb + 1/2 (Va - Vb) // Simplify (* Output is averge of Va and Vb *)
```

$$\frac{Va + Vb}{2}$$

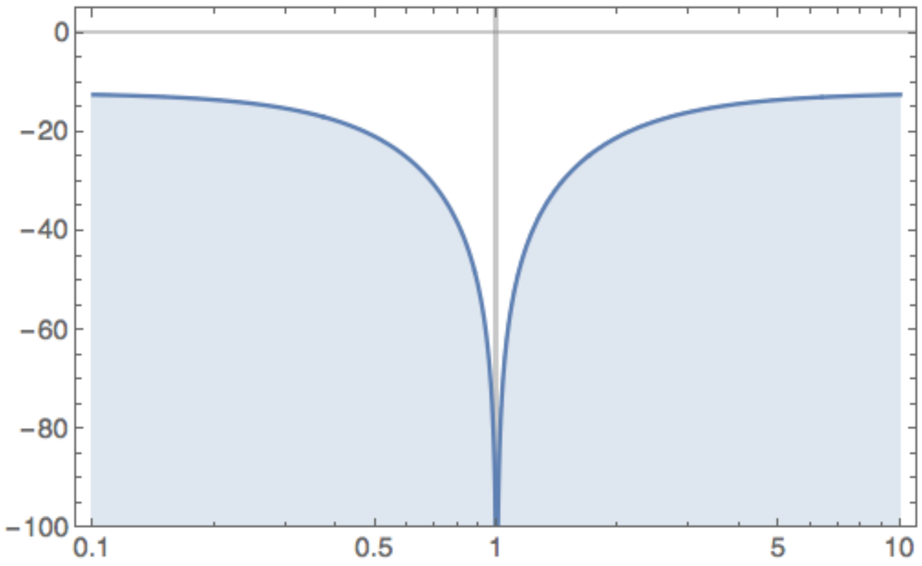
```
H = (HLP HLP + HHP HHP)/2 // Simplify
```

$$\frac{-1 + RC^2\,\omega^2}{2\,(-i + RC\,\omega)^2}$$

```
HMag = H Conjugate[H] /. RC → 1 // FullSimplify
```

$$\frac{(-1 + \omega^2)^2}{4\,(1 + \omega^2)^2}$$

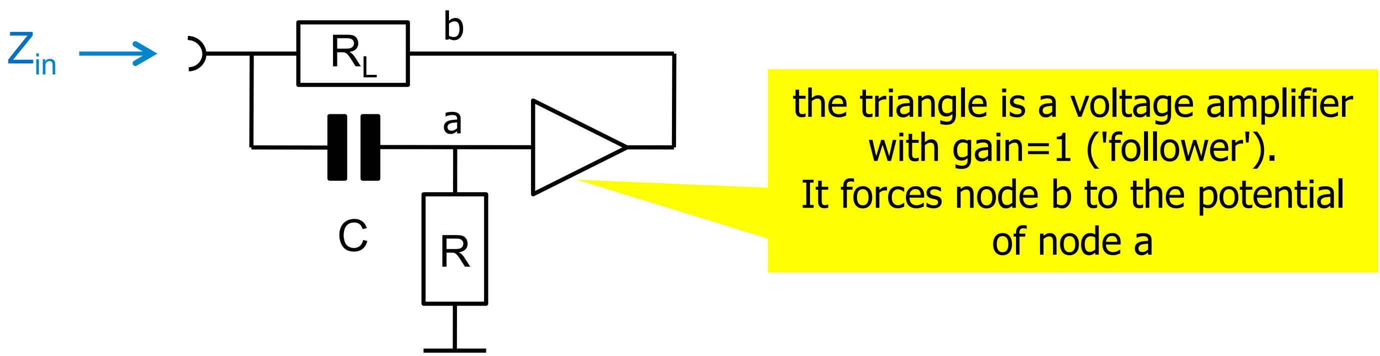
```
LogLinearPlot[dB[HMag], {ω, 0.1, 10}
 , GridLines → {{1}, {0}}, PlotRange → {-100, 5}, Filling → -100]
```



- At the corner frequency, the signal is fully stopped!
- This is because **the phases** of the two signals are ± 90°, i.e. the signals are complementary
  - (A bit tricky to verify in Mathematic due to jump in ArcTan[]..)

Exercise 1.6: Gyrator (difficult)

- A 'Gyrator' can mimic inductive behaviour, while using only resistors, capacitors and amplifiers
- Consider the following circuit:



- **Calculate** the input impedance $Z_{in} = U_{in}/I_{in}$ of the circuit
  - (Use Kirchhoff's law at the input node and node a)
- For frequencies < 1/C R_L, the denominator can be neglected.
- Compare the result to an inductor in series with R_L
- Simulate.
  - Note that R should be larger than R_L (what happens for $R=R_L$?)
  - Plot i_{in}.
  - Add another capacitor in series to produce a resonant circuit.

Solution 1.6

- Mathematica:

```
EQin = iin == (vin - va) s C + (vin - vb)/RL /. vb → va;
EQa = (vin - va) s C == va / R;
Eliminate[{EQin, EQa}, va] // Simplify
```

$iin\,(RL + C\,R\,RL\,s) == vin + C\,RL\,s\,vin$

```
sol = Solve[%, iin] // First
```

$\left\{iin \to \frac{vin + C\,RL\,s\,vin}{RL\,(1 + C\,R\,s)}\right\}$

```
Zgyrator[s_] = vin/iin /. sol // Simplify
```

$\frac{RL + C\,R\,RL\,s}{1 + C\,RL\,s}$