Exercise:
(More) Filters

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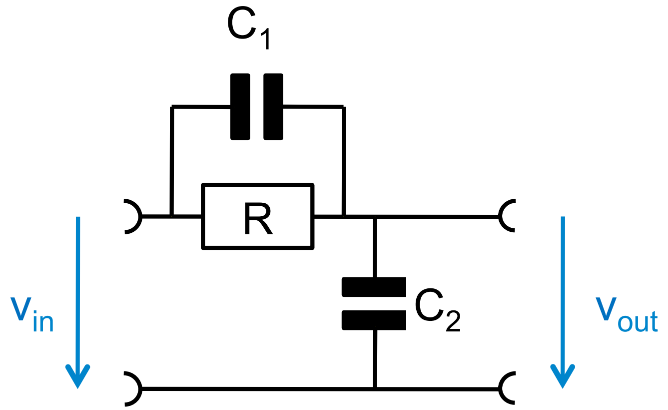
Exercise 1

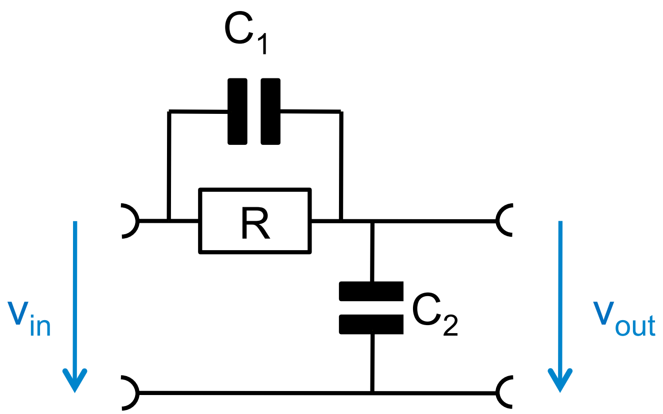
- Analyze the following circuit (simulation & calculation!):



- What is the transfer function ?
- At which frequencies are the 'pole' in the denominator and the 'zero' in the nominator ?
- What are gain and phase for $s \rightarrow 0$ and for $s \rightarrow \infty$? Why?
- What happens for $C_1 \rightarrow 0$, for $R \rightarrow 0$, for $R \rightarrow \infty$? Reasonable?
- Simulate the circuit for $C_1 = C_2 = 10$pF and R = 10 kΩ. Plot gain and phase!
- Chose values so that the circuit attenuates to 1/10 at high frequencies.
- For fun: At which frequency is phase shift maximal?

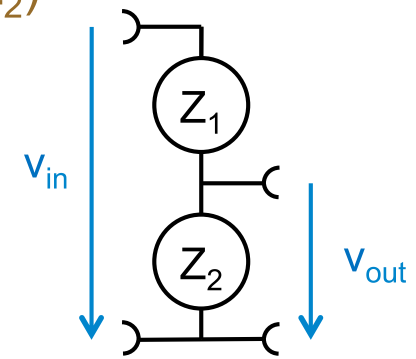
Solution 1

- Two possibilities:
  1. Treat circuit as voltage divider with C_1 // R and C_2
  2. Use Kirchhoff's law @ node v_{out}



- Voltage divider:
  - For any Z_1, Z_2, we have $v = v_{out}/v_{in} = Z_2 / (Z_1+Z_2)$
  - With $1/Z_1 = 1/R + s\,C_1$ and $1/Z_2 = s\,C_2$:

$$v = \frac{1 + C_1 R s}{1 + (C_1 + C_2) R s}$$



- Kirchhoff:
  - Solve $(v_{in}-v_{out})/R+(v_{in}-v_{out})\,sC_1 = v_{out}\,s\,C_2$ for v_{out}

Solution 1

$$v = \frac{1 + C_1 R s}{1 + (C_1 + C_2) R s}$$

- Limits
  - s → 0: caps are gone. v is just 1. No phase shift.
  - s → ∞: R can be neglected. frequency dependencies cancel. This is just a capacitive voltage divider. No phase shift
- Phase shift:

```
gain = Sqrt[(HH /. s → ⅈ ω) (HH /. s → -ⅈ ω)] // FullSimplify
```

$$\sqrt{\frac{1 + C1^2 R^2 \omega^2}{1 + (C1 + C2)^2 R^2 \omega^2}}$$

```
phase = -180/π ArcTan[Im[ComplexExpand[HH /. s → ⅈ ω]] / Re[ComplexExpand[HH /. s → ⅈ ω]]] // FullSimplify
```

$$\frac{180 \,\text{ArcTan}\left[\frac{C2\, R\, \omega}{1 + C1\,(C1 + C2)\, R^2 \omega^2}\right]}{\pi}$$

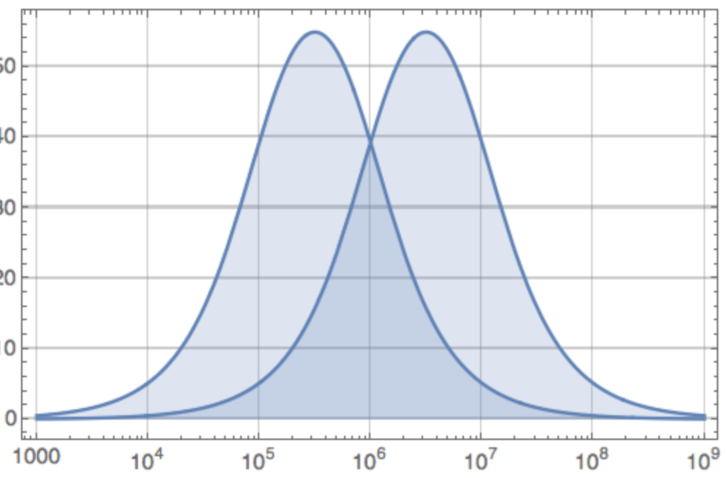
```
PhaseDeriv = D[phase, ω] // FullSimplify
```

$$\frac{180\, C2\, R\left(1 - C1\,(C1 + C2)\, R^2 \omega^2\right)}{\pi + \left(2\, C1^2 + 2\, C1\, C2 + C2^2\right) \pi R^2 \omega^2 + C1^2\,(C1 + C2)^2 \pi R^4 \omega^4}$$

```
ωmax = ω /. Solve[PhaseDeriv == 0, ω] // Last // FullSimplify
```

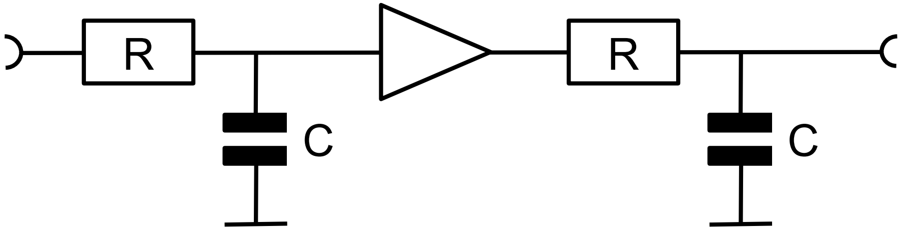
$$\frac{1}{\sqrt{C1\,(C1 + C2)\, R^2}}$$

```
LogLinearPlot[phase /. PARAM, {ω, 1×10^3, 1×10^9}]
```



Exercise 2: Cascaded Stages

- Consider the following two stage circuit (again):



- The triangle is a (voltage) buffer with infinite input impedance (it does not load the first low-pass) and zero output impedance. From the analogLib, use vcvs (voltage controlled voltage source)
- What transfer function do you expect ?
- Simulate the circuit !
- **Simulate** *in parallel* a version **without** buffer.
  Where are differences ?
- Use a much larger R and correspondingly smaller C in the second low pass.
- Now **calculate** the exact transfer function **without** buffer

Solution 2

- Transfer Function with buffers

```
HH1 [s_] = (1/(1 + s R C))^2; (* with buffers: square of s
gain1[ω_] = Sqrt[HH1[ⅈ ω] HH1[-ⅈ ω]] // FullSimplify
```

$$\frac{1}{1 + C^2 R^2 \omega^2}$$

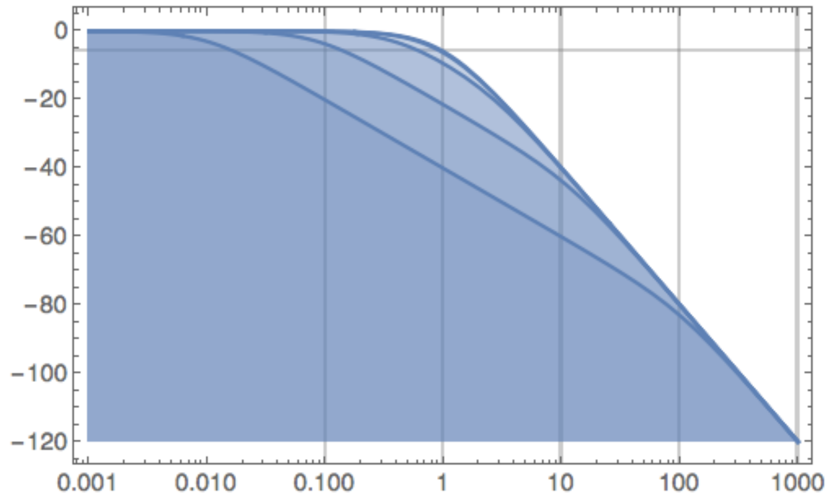
- Without Buffers:

```
EQ1 = (vin - v1)/R1 == v1 s C1 + (v1 - vout)/R2; (* node v1 *)
EQ2 = (v1 - vout)/R2 == vout s C2; (* output node *)

Eliminate[{EQ1, EQ2}, v1] // Simplify
vin == (1 + C2 (R1 + R2) s + C1 R1 s (1 + C2 R2 s)) vout

HH2 [s_] = vout/vin /. Solve[%, vout] // First
```

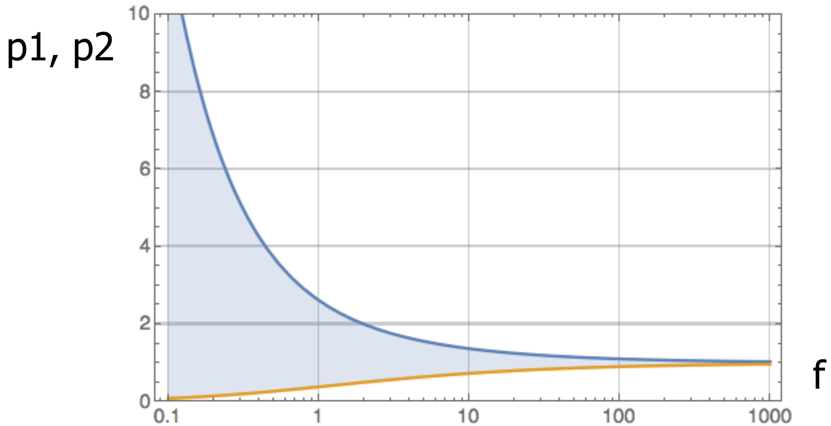
$$\frac{1}{1 + C1\,R1\,s + C2\,R1\,s + C2\,R2\,s + C1\,C2\,R1\,R2\,s^2}$$

Solution 2

- Bode Plot for different RC combinations in second stage:
  - We note two poles.
  - They coincide, when the second low pass does not load the first

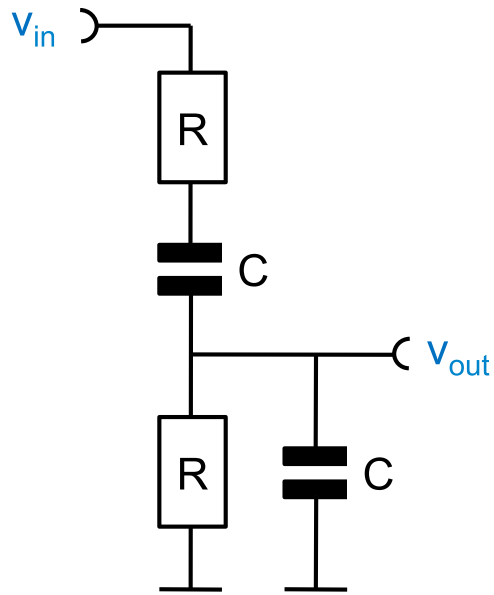


- Plot the poles as a function of *f* where $R_2 = R_1 f$, $C_2 = C_1 / f$, so that $R_2C_2 = R_1C_1$:



Exercise 3: Wien Bridge / Oscillator

- Consider this circuit:



- What is the transfer function?
- What is the magnitude at the center frequency?
- What is the Phase at the center frequency?
- Simulate the circuit for R=1k C=1n

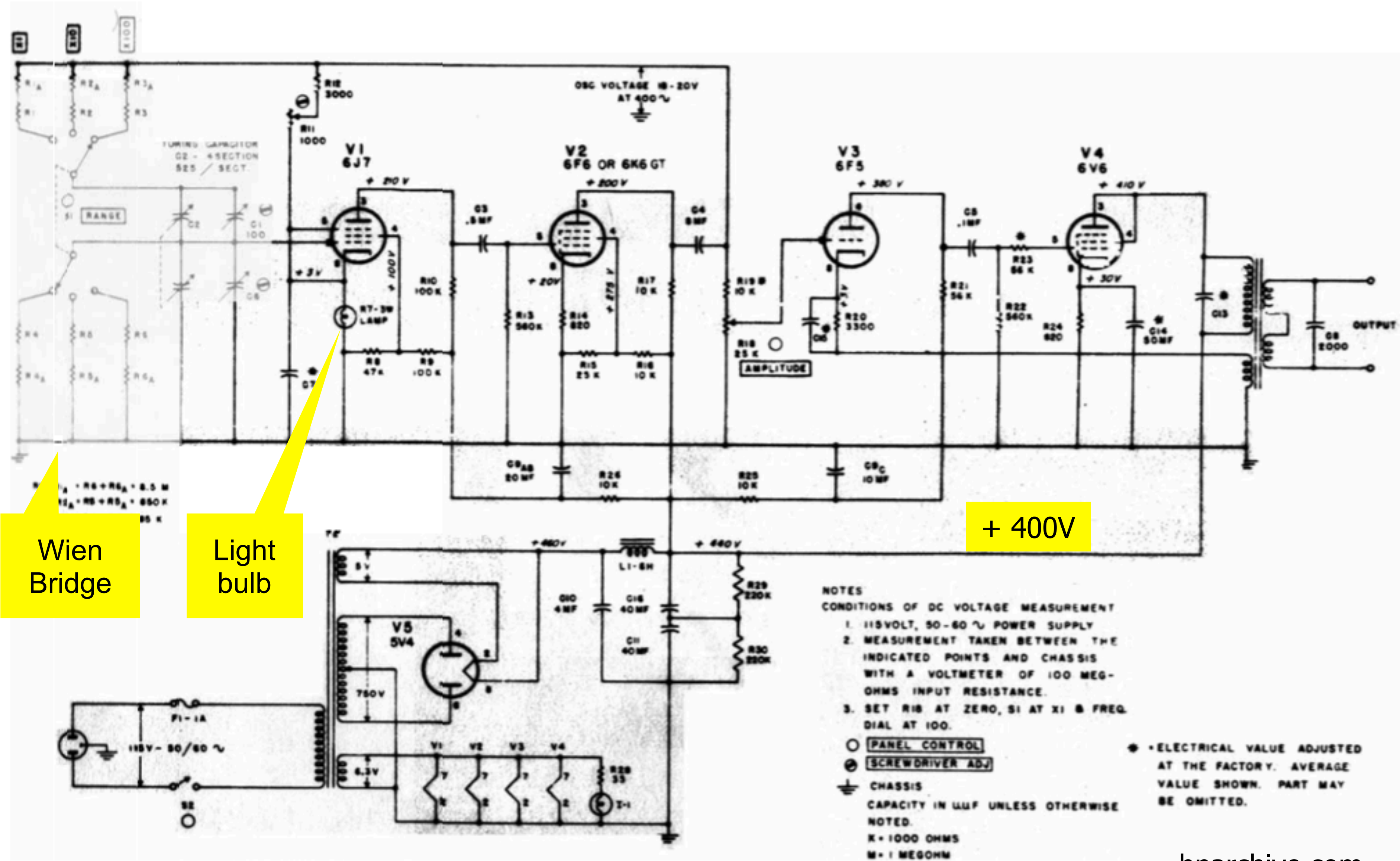
- You can use this 'Wien Bridge' to make an oscillator:
  - Amplify v_{out} by *exactly* 3 (vcvs !) and feed the signal back to v_{in}.
  - Set an initial condition of 1V (parameter!) for the lower C and start a transient simulation.

  - How does this work?
  - What happens if the gain is not exactly 3 ?

Itermezzo: Wien Oszillator

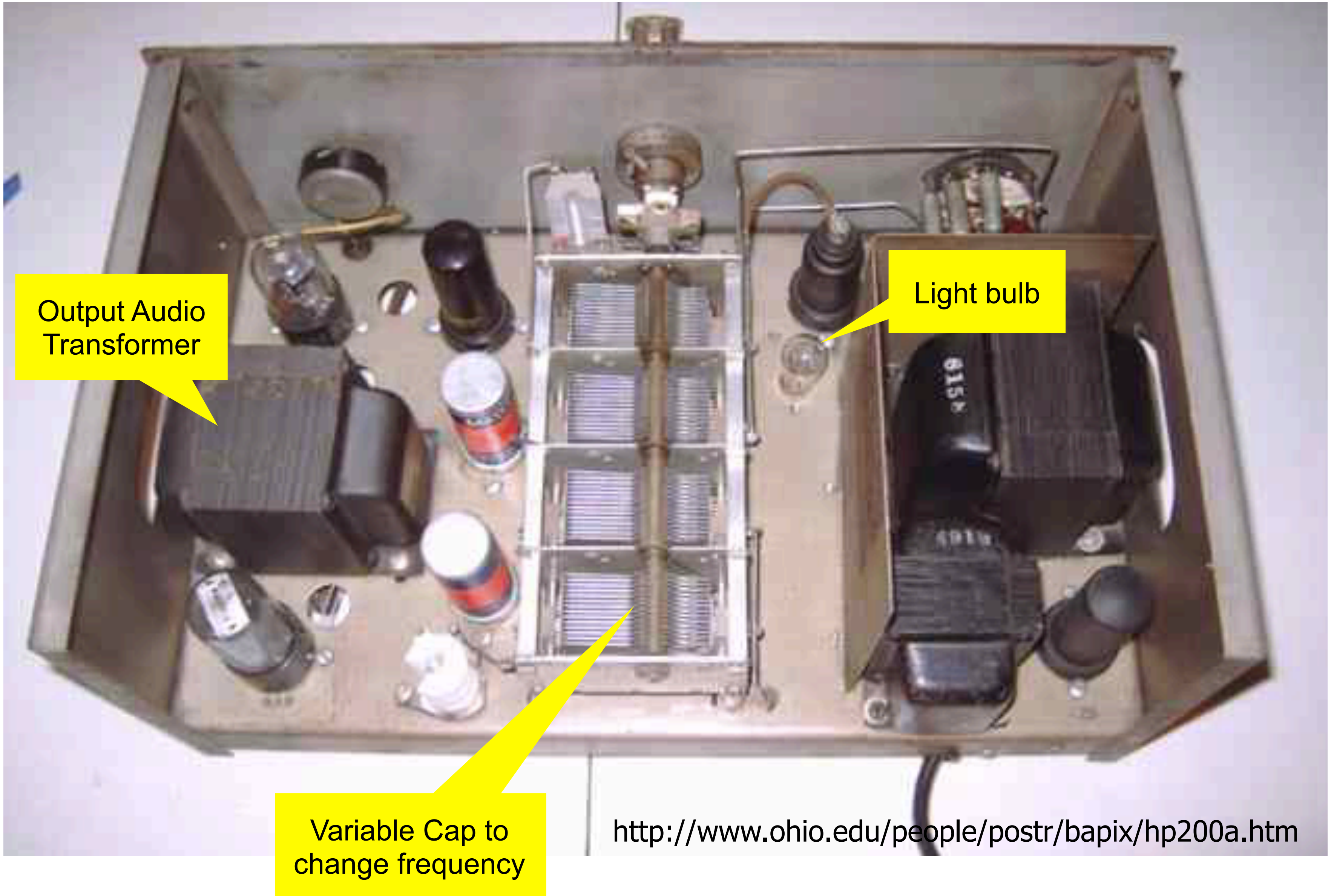
- Wien bridge: Max Wien (1891)
- An Oscillator using a *light bulb* to stabilize gain (leading to with very low distortion) is patented 1939 by William Hewlett and David Packard (Stanford University)
- Founders of Hewlett – Packard (HP)
- Their first product: HP 200A 'Audio Oscillator'

Schematic Diagram (HP 200 B)



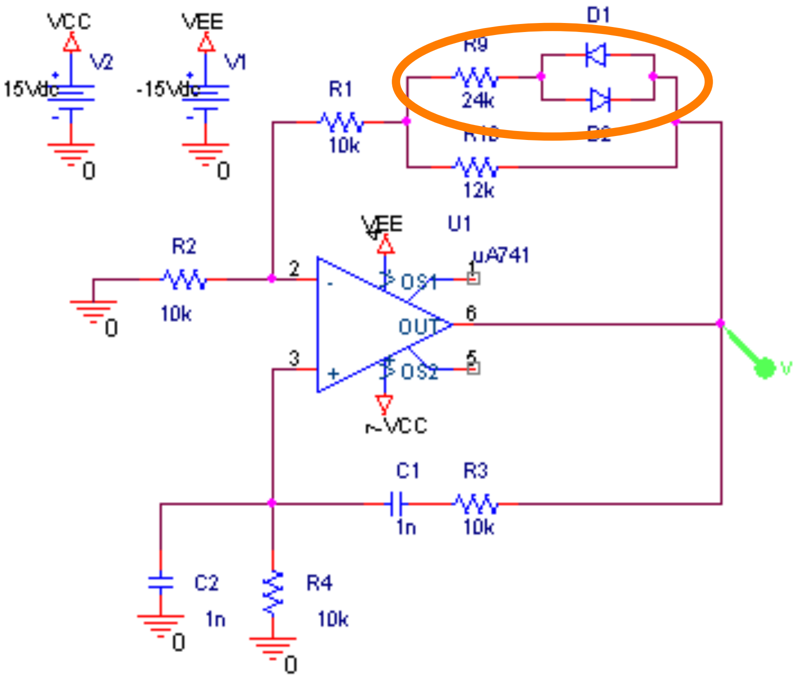
hparchive.com

Inside..

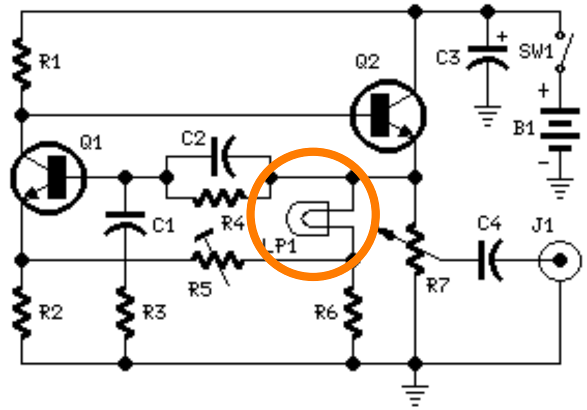

Output Audio Transformer
Light bulb
Variable Cap to change frequency
http://www.ohio.edu/people/postr/bapix/hp200a.htm

Discrete circuit versions

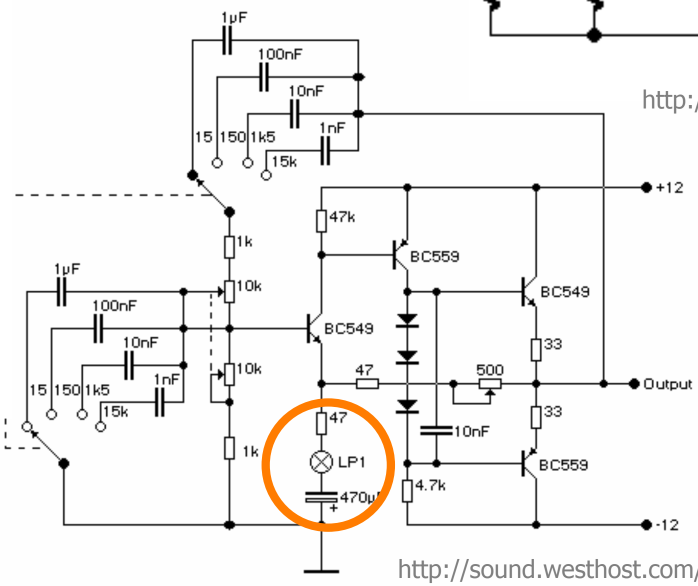
- The problem is to stabilize the gain to exactly 3
- This is achieved by a regulation loop which monitors the output amplitude by some means



www.calvin.edu/~pribeiro/courses/engr332/Handouts



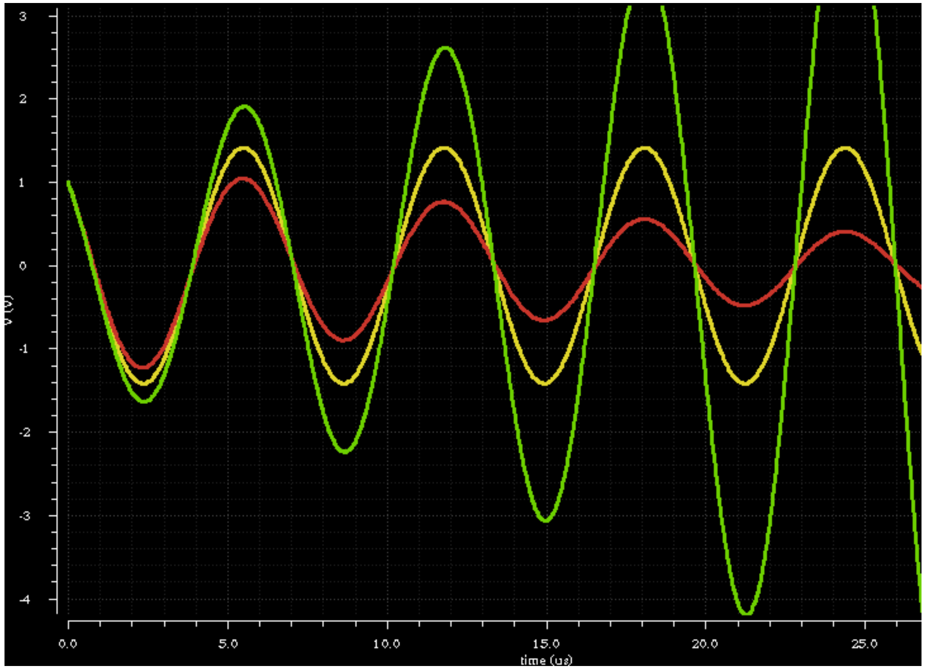
http://www.redcircuits.com//Page13.htm



http://sound.westhost.com/project22.htm

Solution 3

- $H[s] = \dfrac{C R s}{1 + 3 C R s + C^2 R^2 s^2}$
- Centre frequency is at $\omega_0 = 1/RC$.
  - Gain there is 1/3. Phase is 0
  - Note that H becomes real valued at $s = i\,\omega_0$
- Oscillator:



Exercise 5: Low Pass & High Pass

- Plot (on a paper) the Bode diagram for a low pass filter
  - **Calculate** the amplitude at the corner frequency $\omega_C = 1/(RC)$
  - **Calculate** the phase for $\omega = 0$, $\omega = \omega_C$, $\omega \to \infty$

- Simulate this circuit (using an ac sweep)
  - Do not forget to set the 'ac magnitude' to 1
  - Change the vertical scale to 'log'
  - Plot gain in dB and phase:
    Results → Direct Plot → Gain and Phase → Select output and the Input
  - Check the numerical value of the corner frequency

- Add a high pass and plot the two outputs simultaneously