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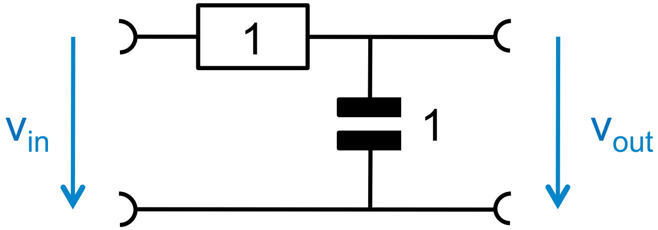
For Fun:
Higher Order Filters

(Mathematica file: CCS_HigherOrderFilters.nb)

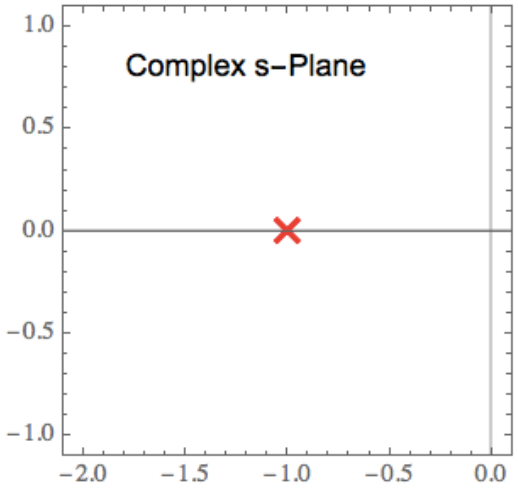
Reminder: One Low Pass

- (For simplicity, we use fixed values for R and C, often 1 Ω/F)



$$H_{LP1}[s\_] = \frac{1}{1+s}$$

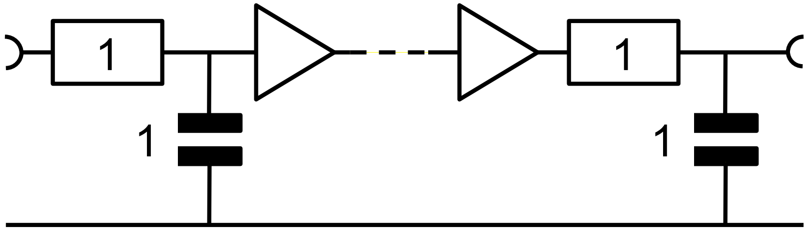
- Mathematically, $H_{LP1}[s]$ has a ***POLE*** at s = -1.
- This can be illustrated in the COMPLEX s-Plane:



- This particular pole is ***real***, i.e. it lies on the real axis

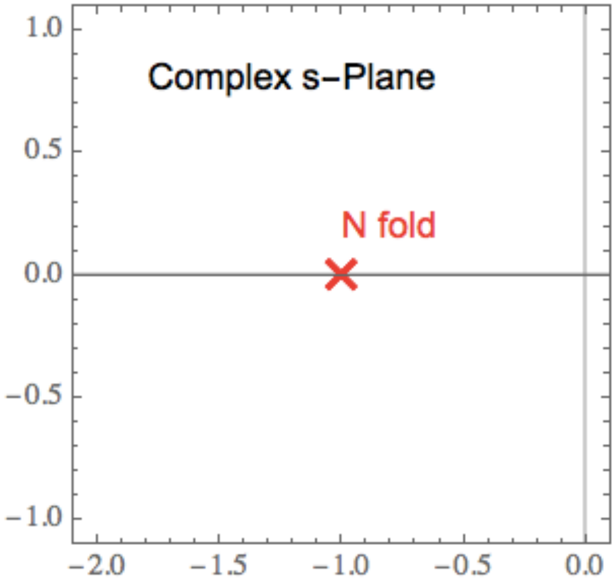
Reminder: Cascaded Low Pass Stages

- If we cascade N stages *with buffers*, we get



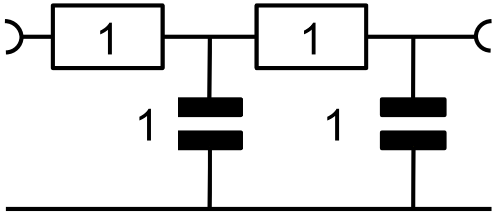
$$H_{LPN}[s\_] = \frac{1}{(1+s)^N}$$

- $H_{LPN}[s]$ has a ***N-fold*** POLE at the same location s = -1.



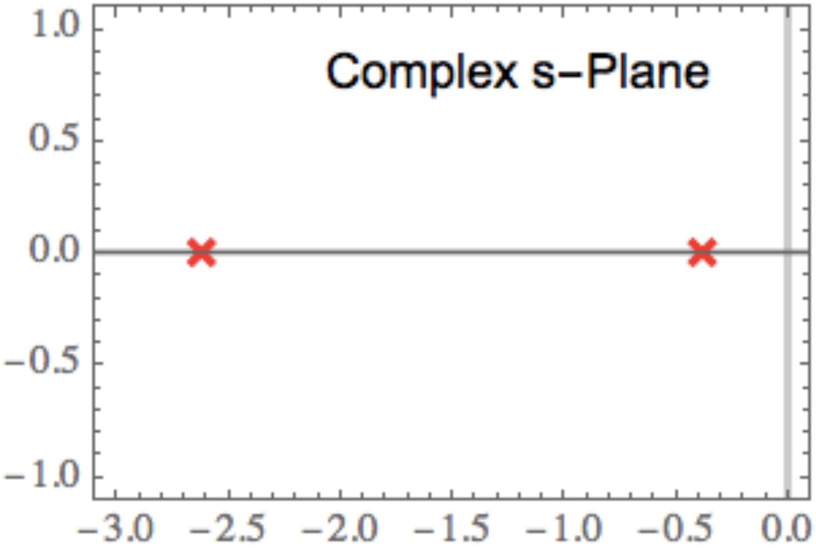
Two Unbuffered Low Pass Stages

- If we cascade two stages *without buffer*, we get



$$H_{LPCasc}[s] = \frac{1}{1 + 3\,s + s^2}$$

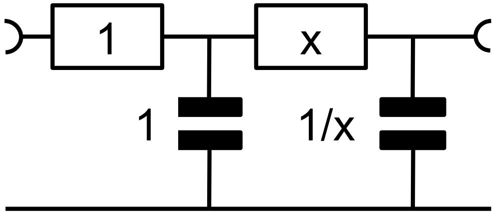
- We now have ***two different*** (still real) poles:



(Their locations depend on R/C of the second stage)

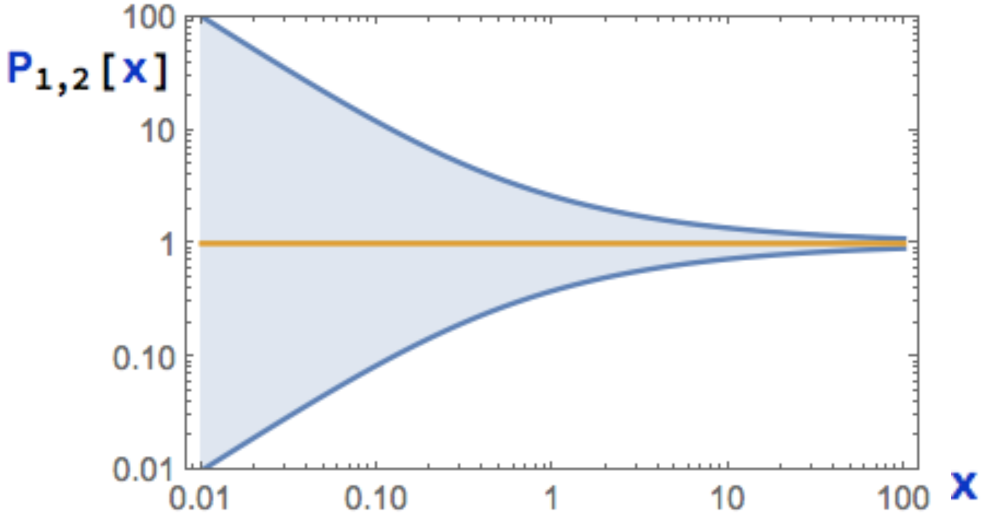
(Pole Location for Previous Case)

- If we modify R,C of the second stage, keeping RC =1, we get



$$H[s\_] = \frac{x}{s + x + 2\,s\,x + s^2\,x}$$

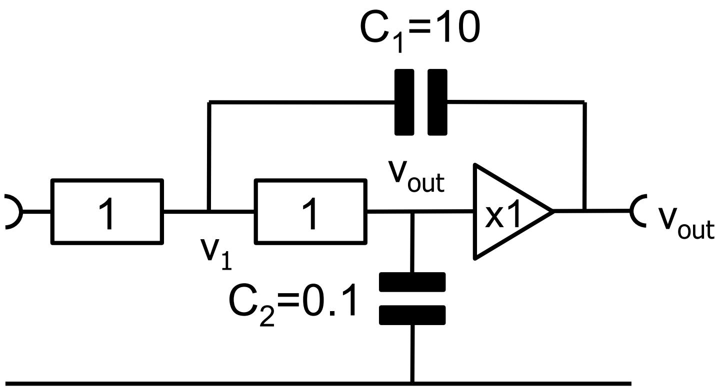
- The poles are at $P_{1,2}[x] = \dfrac{-1 - 2\,x \pm \sqrt{1 + 4\,x}}{2\,x}$



(when x is large, the 2nd LP does not load the 1st)

An Active Filter

- Now consider the following filter ('Sallen and Key')

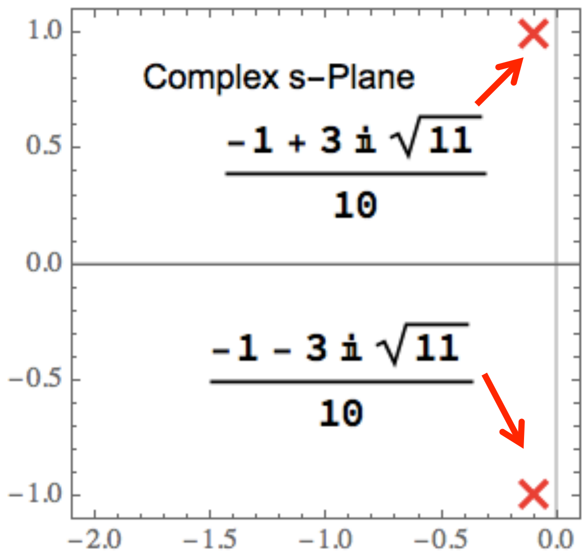


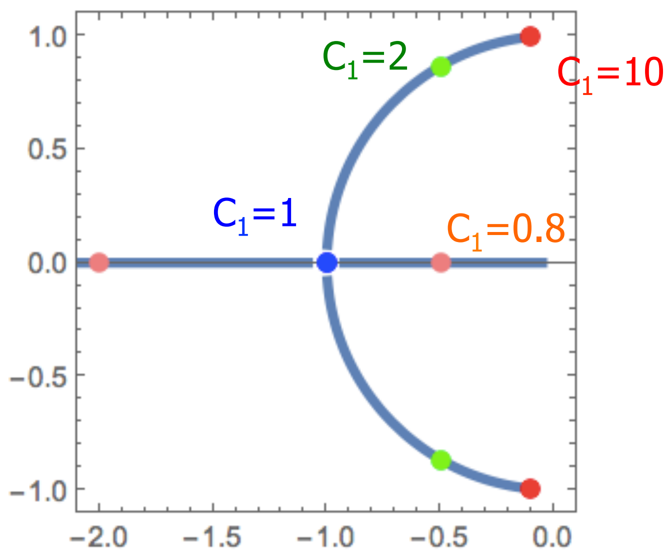
$$EQ1 = \frac{vin - v1}{1} == \frac{v1 - vout}{1} + (v1 - vout)\, s\, 10;$$

$$EQ2 = \frac{v1 - vout}{1} == vout\, s\, \frac{1}{10};$$

$$H[s] = \frac{5}{5 + s + 5 s^2}$$

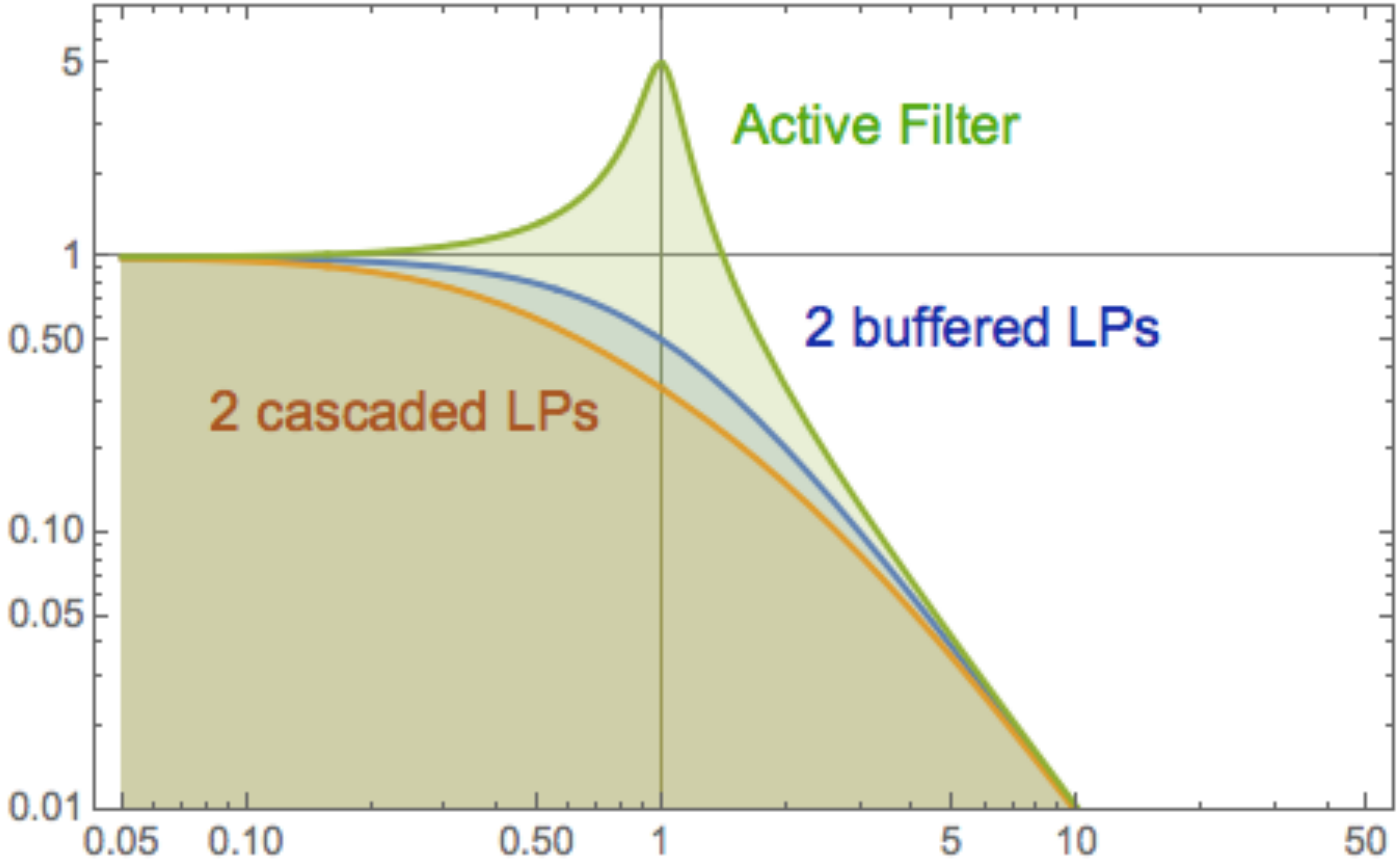
- This transfer function has two ***COMPLEX*** (conjugate) poles:





Bode Plots of 2nd Order Filters

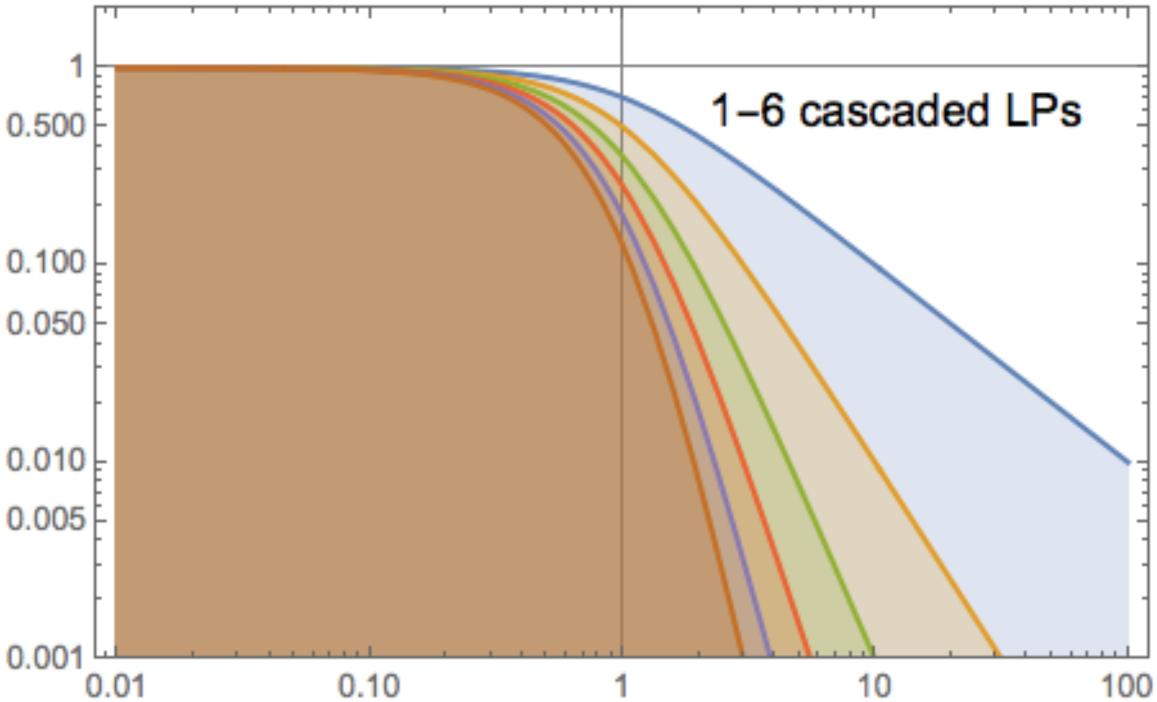
- The active filter has an overshoot (for the values chosen)
- This is typical for complex conjugate poles



Making Steep Filters

A Steep Low Pass Filter

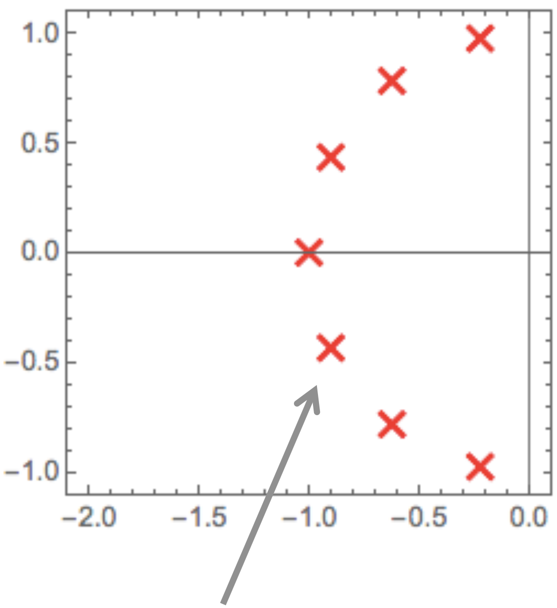
- We want to design a higher (N^{th}) order low-pass filter which drops suddenly from **pass band** to **stop band**.
- We know that we roll off with slope –N at the end (for s→∞).



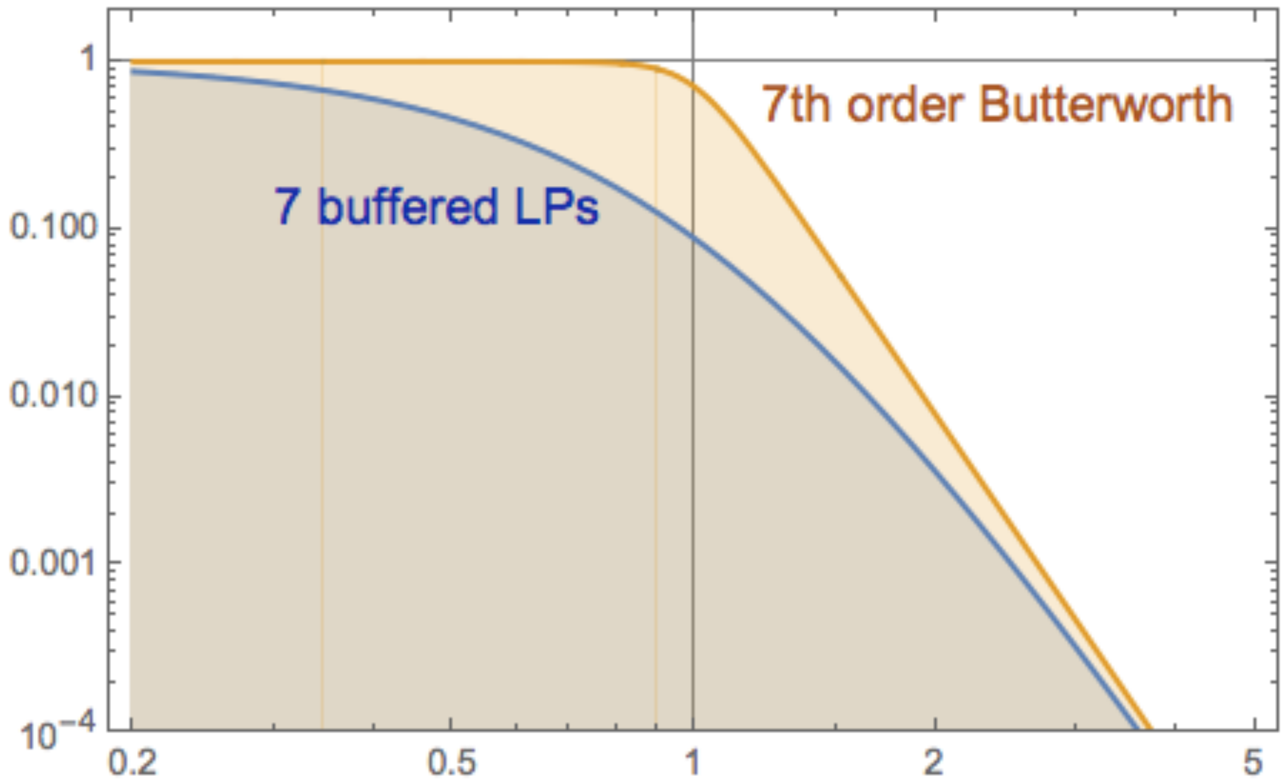
- Simple cascaded LPs attenuate by $2^{-N/2}$ at the corner
- Can this be improved ?

Choosing the Poles

- The Idea: Use complex poles and adjust them 'somehow'
- 'Butterworth' arranges poles on circle. Here: 7th order.



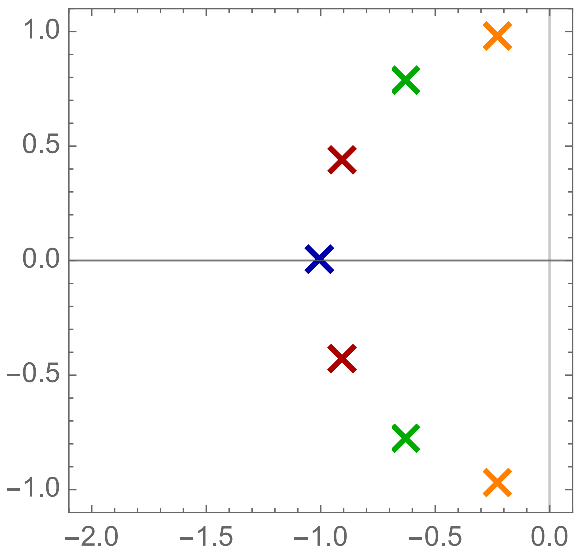
It can be shown (easily) that poles on a circle have same corner frequencies.

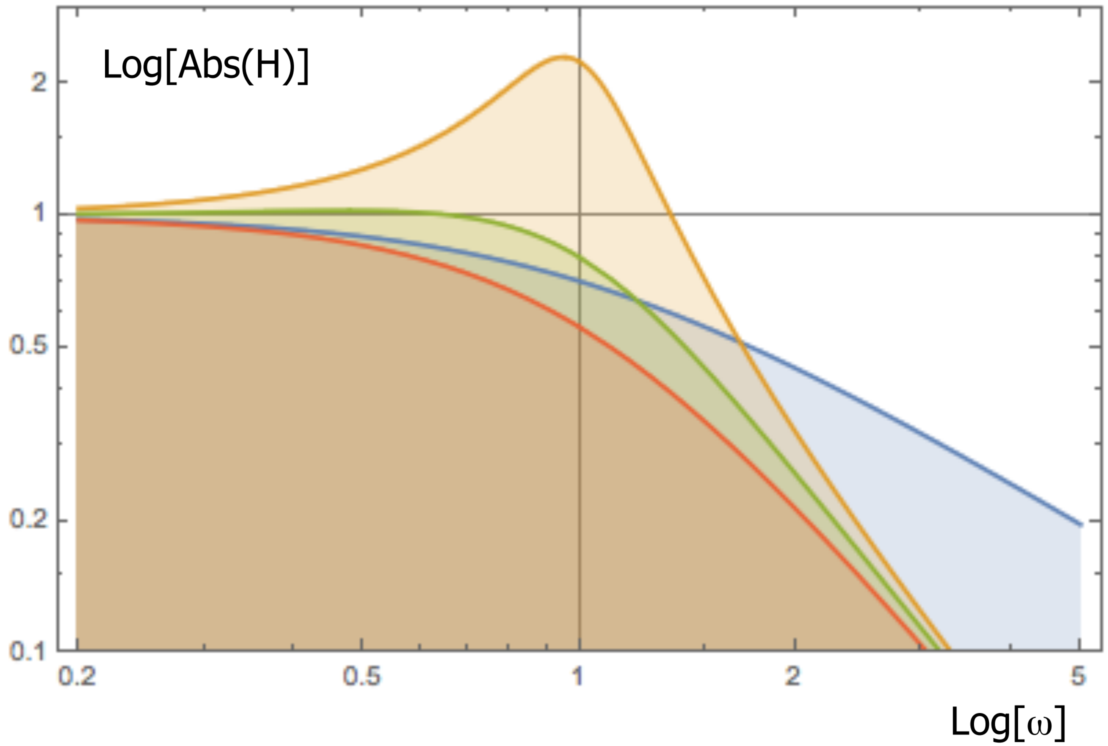


- Wow! Butterworth attenuation at the corner is only -3dB !

(Decomposing the Butterworth Filter)

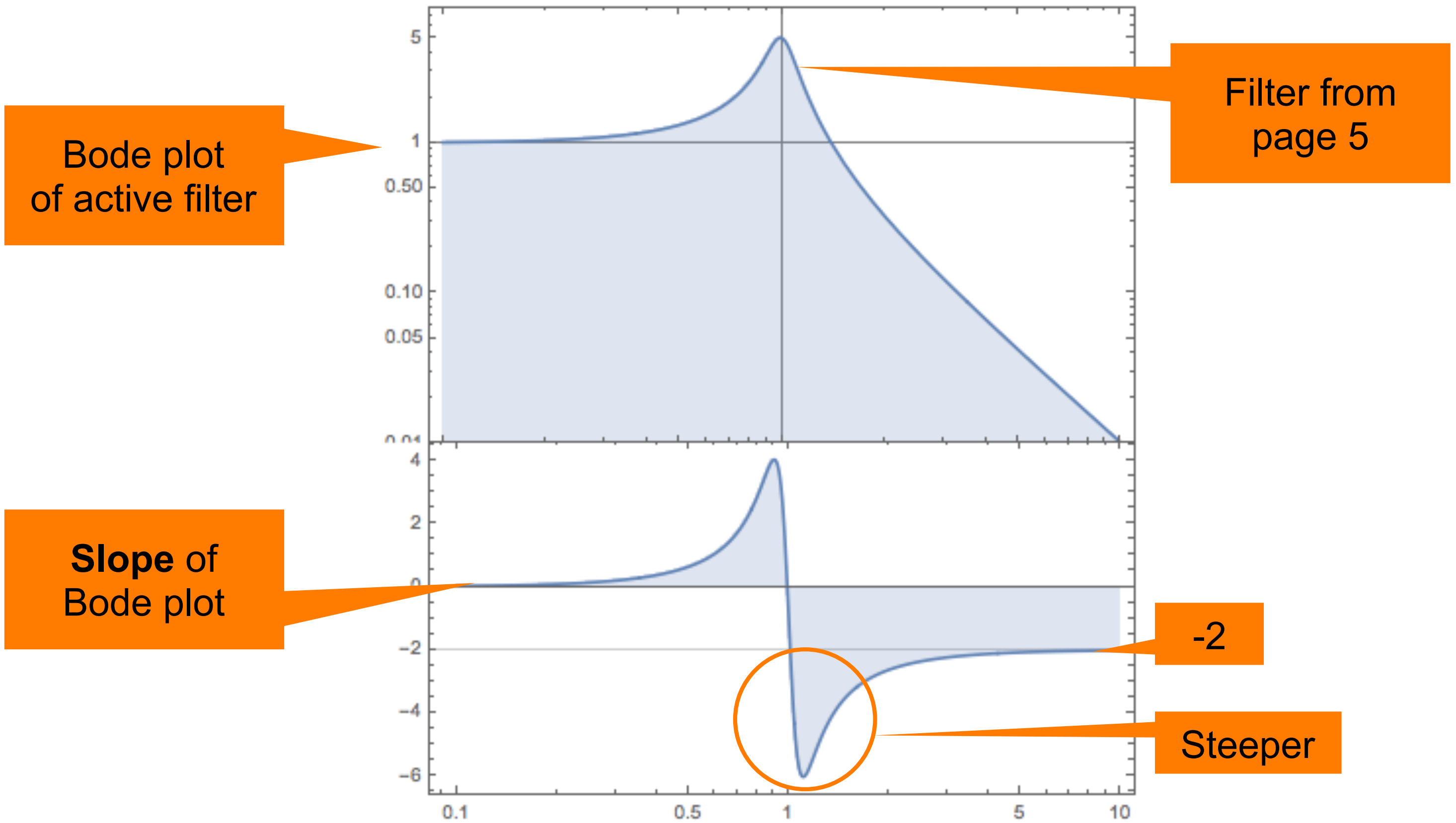
- For N=7:
  - One real pole (1st order, blue)
  - 3 conjugate poles (2nd order)





Even Steeper?

- Remember: For *large* frequencies, we will *always* roll off with s^{-N} (the order of the filter, i.e. the number of caps)
- But: The 'peaking' for complex poles provides steeper response *close to the bandwidth*:



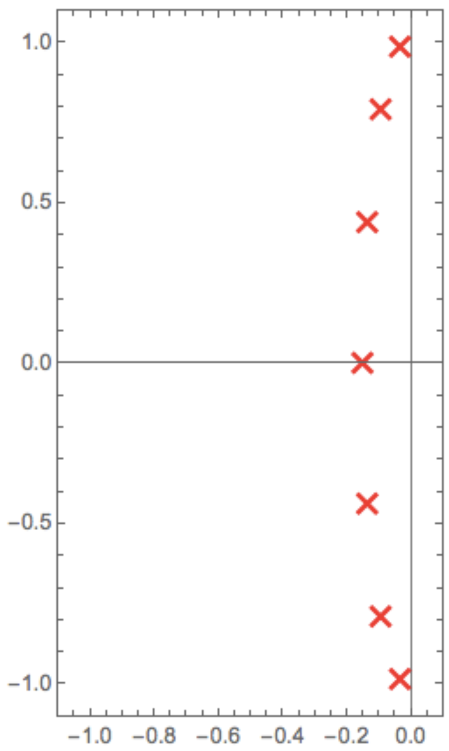
Placing the Poles...

- There are obviously MANY possibilities to place the poles…

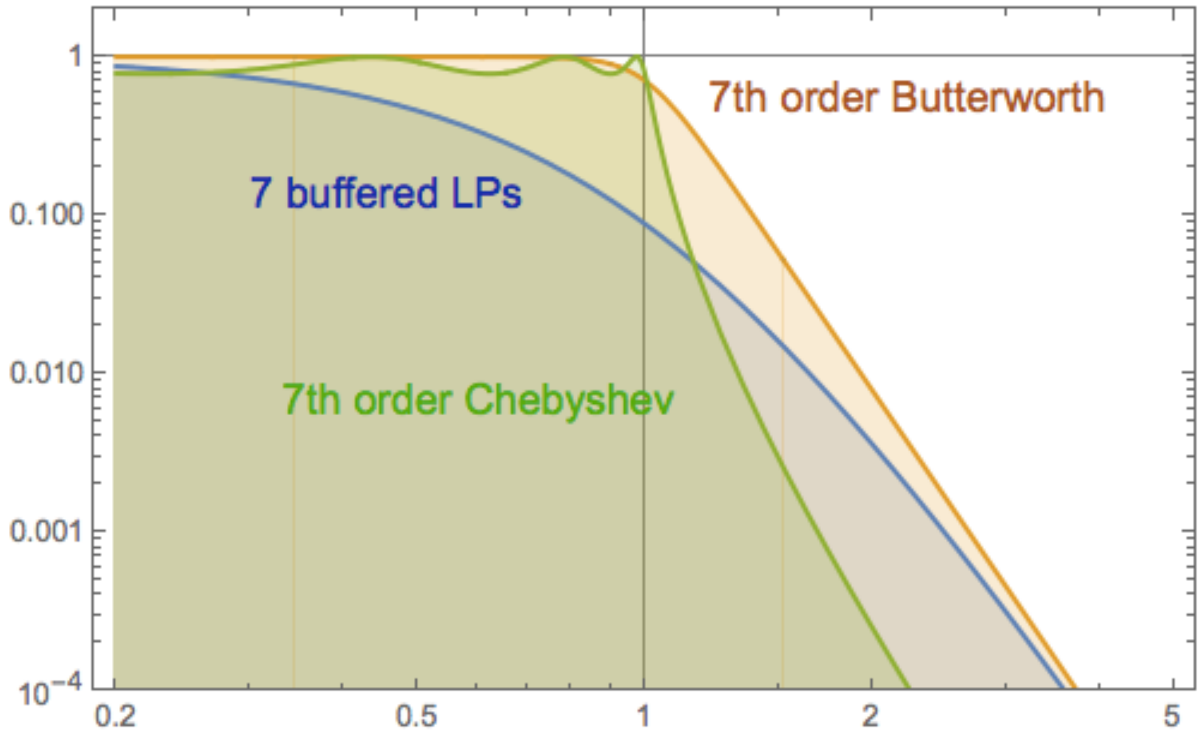
- Desired filter properties are for instance
  - Flatness/ripple of the response in the pass band
  - Steepness of the drop
  - Ripple in the stop band
  - Response to step signals (overshoots)
  - Phase behavior

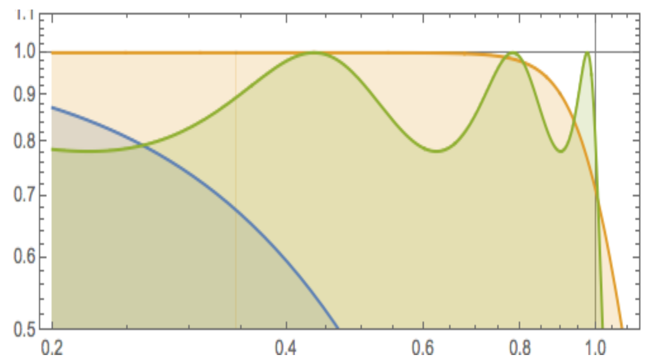
- Four main types have evolved:
  - Butterworth: Flat pass band
  - Bessel: No phase shift, no overshoot
  - Chebyshev: Steeper rolloff, but ripple in pass band
  - Elliptic: Even steeper rolloff, but ripple in pass and stop band

The Chebyshev Filter (7th order)



Pole location for a 7th order Chebyshev filter (there are others, depending on the desired pass band ripple)





Zoom of pass band showing ripple of Chebyshev and flat response of Butterworth