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Exercise:
Transfer Functions, Filters

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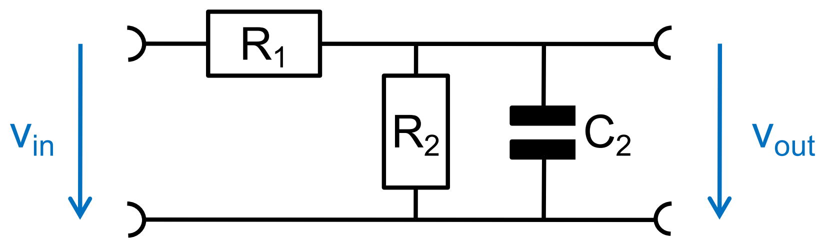
Recommendations

- I strongly recommend to use a mathematical program (Mathematica, Maple, SageMath,..) to solve the exercises

- For transfer functions, inspect each result:
  - What happens for $\omega \to 0,\infty$?
  - What happens if component values go to 0 or ∞?

Exercise 1

- Derive the Transfer Function of this circuit:

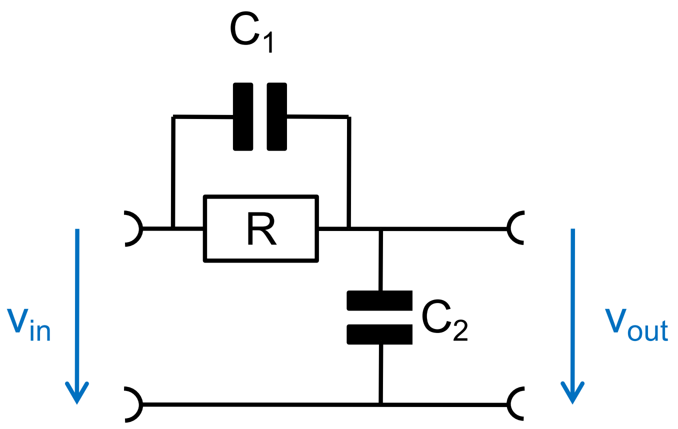


- Use 3 different approaches:
  - Treat the circuit directly (using Kirchhoff's rule)
  - Consider it as a voltage divider of two Impedances. Use R_1 for Z_1 and the parallel connection of R_2 and C_2 for Z_2
  - Replace the (resistive) voltage divider (R_1,R_2) by its Thévenin equivalent and then add the capacitor

- Make a Bode Plot
  - Observe the difference to the normal Low Pass Filter

Exercise 2

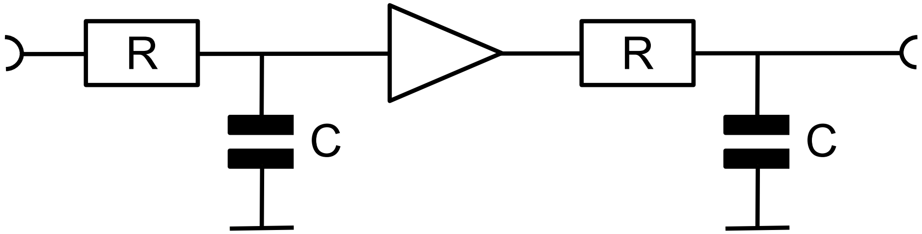
- Analyze the following circuit (simulation & calculation!):



  - What is the transfer function ?
  - At which frequencies are the 'pole' in the denominator and the 'zero' in the nominator ?
  - What are gain and phase for $s \rightarrow 0$ and for $s \rightarrow \infty$? Why?
  - What happens for $C_1 \rightarrow 0$, for $R \rightarrow 0$, for $R \rightarrow \infty$? Reasonable?
  - Simulate the circuit for $C_1 = C_2 = 10\text{pF}$ and $R = 10\ \text{k}\Omega$. Plot gain and phase!
  - Chose values so that the circuit attenuates to 1/10 at high frequencies.
  - For fun: At which frequency is phase shift maximal?

Exercise 3: Cascaded Stages

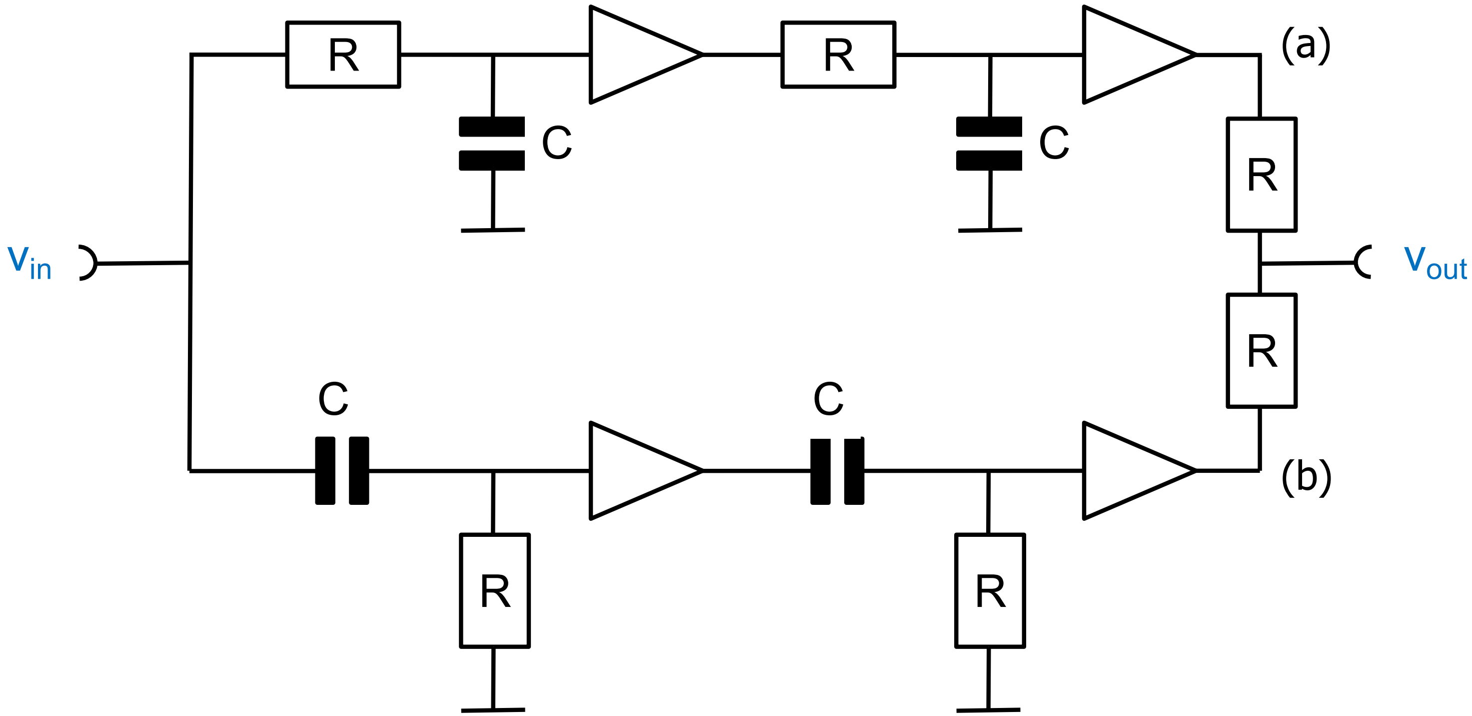
- Consider the following two stage circuit (again):



- The triangle is a (voltage) buffer with infinite input impedance (it does not load the first low-pass) and zero output impedance. For simulation use a vcvs (voltage controlled voltage source) from analogLib with gain 1
- What transfer function do you expect ?
- Simulate the circuit !
- **Simulate** a version **without** buffer in the same schematic
- Where are differences ?
- Use a much larger R and correspondingly smaller C in the second low pass.
- Now **calculate** the exact transfer function **without** buffer

Exercise 4: Notch Filter

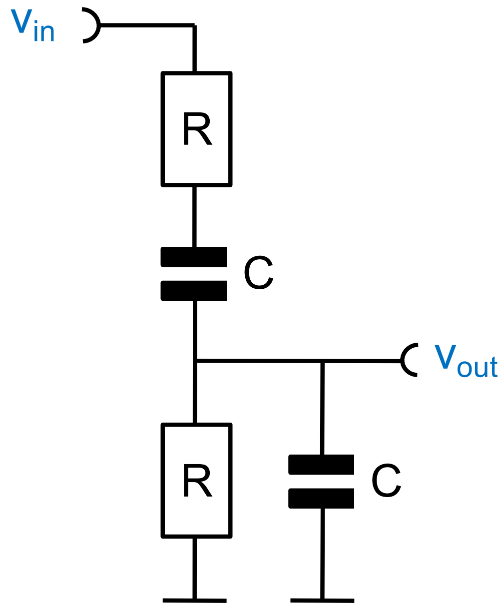
- Consider the following circuit made of cascaded High- and Low Pass stages:
  - The resistors at the output just add the signals at (a) and (b)



- What is the output signal at the corner frequency?
  - Explain this by comparing amplitudes ***and phases*** at (a) and (b)

Exercise 5: Wien Bridge / Oscillator

- Consider this circuit:



- What is the transfer function?
- What is the magnitude at the center frequency?
- What is the Phase at the center frequency?
- Simulate the circuit for R=1k C=1n

- You can use this 'Wien Bridge' to make an oscillator:
  - Amplify v_{out} by *exactly* 3 (vcvs !) and feed the signal back to v_{in}.
  - Set an initial condition of 1V (parameter!) for the lower C and start a transient simulation.
  - How does this work?
  - What happens if the gain is not exactly 3 ?

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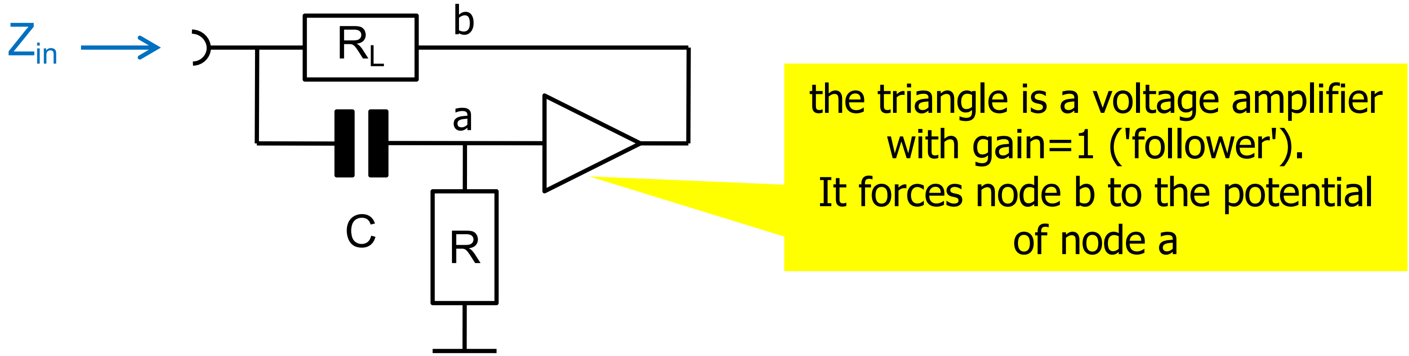
Exercise 6: Gyrator (difficult)

- A 'Gyrator' can mimic inductive behaviour, while using only resistors, capacitors and amplifiers
- Consider the following circuit:



- **Calculate** the input impedance $Z_{in} = U_{in}/I_{in}$ of the circuit
  - (Use Kirchhoff's law at the input node and node a)
- For frequencies < 1/C R_L, the denominator can be neglected.
- Compare the result to an inductor in series with R_L
- Simulate.
  - Note that R should be larger than R_L (what happens for $R=R_L$?)
  - Plot i_{in}.
  - Add another capacitor in series to produce a resonant circuit.