1. Exercise	Silicon Sensors & Readout Electronics WS16/17

1.1 Spatial Resolution of Strip Structures: Binary Readout

A hit occurs in a strip structure at position x (perpenticular to the strips). With the help of the observed signal amplitudes on the various strips, we reconstruct a position x_{rek}. The *reconstruction error* $x_{err} := x - x_{rek}$ is the difference between the two positions. We define the *spatial resolution* as the standard deviation of x_{err}, i.e. $\sigma := \sqrt{\langle x_{err}^2 \rangle}$, assuming that the average reconstruction error is zero. For now, we consider a 'binary' readout where a strip fires or not, i.e. we do not use an analog pulse height information.

1. For very localized ('pointlike') signals, we get a single hit on the corresponding strip. The reconstructed position is obviously the center of that strip. Calculate the spatial resolution for strips of width a. (Write down first an expression for $x_{err}(x)$, calculate the square and average over all possible positions.)

2. Now assume that the signal has a width b so that two strips 'fire' when the hit lies close to the edge of the strip. (To be more precise, we get a double hit (two adjacent strips fire) for hit positions at $x = \frac{a}{2} - \frac{b}{2} \ldots \frac{a}{2} + \frac{b}{2}$.) What is the spatial resolution as a function of a and b?

3. Assume that we can influence the signal width b. What is the *optimal* value for b to reach the best possible resolution? What resolution do we get? How does this compare to the 'narrow signal' case? Can you understand why this value is best?

4. What is the fraction of single/double hits?

1.2 Improving Spatial Resolution using Analogue Information

The spatial resolution of a strip structure with strip pitch a can be improved if an analogue pulse height information is available. This makes interpolation between strips possible. We want to study this with the help of a small simulation program. Use the programming language / environment you like best.
We use the simplifying assumption that a particle at position x deposits a unity charge homogeneously in the range $x \pm \frac{b}{2}$. The charge $S_i(x)$ seen on strip i is just the integral of the signal charge distribution over that strip. For instance, if a strip is hit in the center, all charge is on this central strip (if $a > b$) and the neighbors have zero charge. If the hit is just at the edge between two strips, both get half charge.

1. Show mathematically that a linear interpolation of the strip signals gives a perfect reconstruction of the hit position for $b = a$.

2. The reconstruction quality is degraded in the presence of noise on the strips. Write a small *Monte Carlo* program which simulates the following steps:

   (a) Use a fixed number of strips (6-10 or so). Reserve a variable $S[i], i = 0 \ldots 9$ to store the corresponding strip amplitudes.

   (b) Determine a random hit position x. This position can be restricted to one strip with no loss of generality. If you generate hits on the whole structure, avoid hits too close to the border.

(c) Calculate the signals on all strips for the box charge distribution of width b.

(d) Add *noise* to *all* strip signals. You can use a box distributed noise with an rms of σ (what is the width of the corresponding box?) for simplicity. (Note that noise can be positive or negative.)

(e) Reconstruct the hit as the 'center of gravity' (linear interpolation).

(f) Determine the reconstruction error.

(g) Calculate the rms of the reconstruction error by averaging over many (10 k or so) events.

3. Make the following checks:

   (a) Reconstruction should be perfect for $b = a$ and no noise.

   (b) You should find the binary limit for small signal widths, independent of the noise.

4. Plot or make a table of the resolution as a function of noise for various signal widths b.

5. In a real system, we must *find* where the signals are. This is not a trivial task in the presence of noise! Try two methods:

   (a) Use *all* signals which are above a 'low' threshold S_{min}.

   (b) Search for 'hits' by setting a 'high' threshold. Use the hit strip an the two neighbor strips for reconstruction.

   You can restrict this study to the case $a = b$. Vary the noise. What happens if thresholds are too high or too low? Which method is better?

6. Optional investigation 1:
   Replace the box distributed noise by Gaussian Noise. You can generate normally distributed random values with the clever Box-Muller transform (`http://en.wikipedia.org/wiki/Box-Muller_transform`). Study what happens!

7. Optional investigation 2:
   In a real system, the analogue amplitudes S_i are digitized with a finite resolution. Investigate the consequences of an N Bit digitization.