Solutions to Exercise: Photon Absorption

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Constants and input values. Units for length are µm!

In[48]:=
```
q = 1.6 × 10^-19; (* in Coulomb*)
ϵ0 = 8.854 × 10^-18; (* F/µm *)
ϵSi = 11.9;
μ = 1400 × 10^8;
(* for electroncs, µm^2/Vs *)
UT = 0.0259; (* thermal
 voltage at room temperatur, V *)
ni = 1.45 × 10^10 / 10^12;
```

In[54]:=
```
$Assumptions = λ > 0;
```

In[55]:=
```
LDEF = 6.3 × 10^3 / 10^4;
(* Absorption coefficient,
converted from cm^-1 to µm^-1*)
```

In[56]:=
```
ρ0 = 1000 × 10^4 (* Bulk resistivity 1kΩ*cm,
 in Ohm*µm *);
```

1. Light Loss

Write down expression for Absorption as a function of depth x

```
NP[x_, λ_] = λ Exp[-λ x];
(* Number of photons absorbed
  from depth x to x+dx is NP[x] dx *)
```

In[58]:=

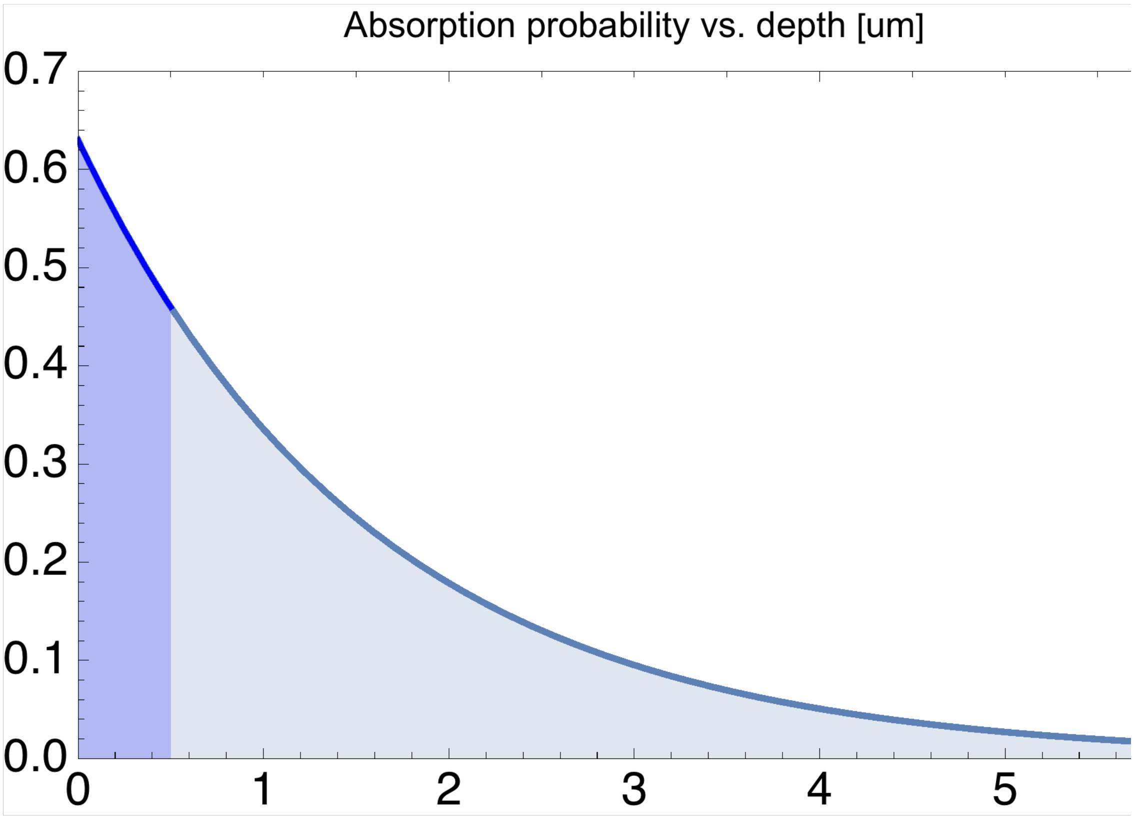
$$\int_0^\infty NP[x, \lambda]\, dx == 1$$

```
(* Check normalization *)
```

Out[58]= True

In[62]:=
```
Plot[
 {NP[x, LDEF], Which[ x < 0.5, NP[x, LDEF]]},
 {x, 0, 6}, PlotRange → {0, 0.7},
 Frame → True, Filling → Axis,
 ImageSize → Medium, PlotLabel →
  "Absorption probability vs. depth [um]"]
```

Out[62]=



In[63]:=
```
Absorption[T1_, T2_, λ_] = ∫_T1^T2 NP[x, λ] ⅆx (* Absorption between T1 to T2 *)
```

Out[63]= $e^{-T1\,\lambda} - e^{-T2\,\lambda}$

In[64]:=
```
Absorption[0, ∞, λ] // Simplify
```

Out[64]= 1

In[65]:=
```
DeadLoss = Absorption[0, 0.5, LDEF]
  (* loss in dead layer *)
```
Out[65]= 0.270211

2. Voltage to get α = 90 % of remaining photons (between T1 and TX)

In[66]:=
```
Absorption[T1, TX, λ] == α * (1 - Absorption[0, T1, λ])
```
Out[66]= $e^{-T1\,\lambda} - e^{-TX\,\lambda} == e^{-T1\,\lambda}\,\alpha$

In[67]:=
```
Solve[%, TX] // First
```
Out[67]= $\left\{TX \to \text{ConditionalExpression}\left[\frac{2\,i\,\pi\,C[1] + \text{Log}\left[-\frac{e^{T1\,\lambda}}{-1+\alpha}\right]}{\lambda},\ C[1] \in \mathbb{Z}\right]\right\}$

In[68]:=
```
% /. C[1] → 0
```
Out[68]= $\left\{TX \to \frac{\text{Log}\left[-\frac{e^{T1\,\lambda}}{-1+\alpha}\right]}{\lambda}\right\}$

In[69]:=
```
ReqDepth = TX /. %
```
Out[69]= $\frac{\text{Log}\left[-\frac{e^{T1\,\lambda}}{-1+\alpha}\right]}{\lambda}$

In[70]:=
```
RequiredDepth =
 ReqDepth /. {λ → LDEF, α → 0.9, T1 → 0.5}
  (* calculate result for
   values in exercise in um *)
```
Out[70]= 4.1549

Calculate the required voltage to get depletion depth ‘RequiredDepth’

In[71]:= $ND = \frac{1}{q\,\rho bulk\,\mu}$ (* calculate n-doping of the bulk from the resistivity *)

Out[71]= $\frac{4.46429\times10^{7}}{\rho bulk}$

In[72]:= $ND * 10^{12}$ /. $\rho bulk \to \rho 0$ (* convert μm^{-3} to cm^{-3} to check that the value makes sense *)

Out[72]= 4.46429×10^{12}

In[73]:= $TDep = Limit\left[\sqrt{\frac{2\,\epsilon 0\,\epsilon Si}{q}\,\frac{NA+ND}{NA\,ND}\,VBi}\,\sqrt{1+\frac{VDep}{VBi}},\ NA\to\infty\right]$ (*Limit of large NA *)

Out[73]= $0.00543153\,\sqrt{\frac{VBi+VDep}{VBi}}\,\sqrt{VBi\,\rho bulk}$

In[74]:= TDep = Assuming[VBi > 0, Simplify[%]] /. VBi + VDep → Vtotal

Out[74]= $0.00543153\,\sqrt{Vtotal}\,\sqrt{\rho bulk}$

In[75]:= Solve[RequiredDepth == TDep, Vtotal] // First

Out[75]= $\left\{Vtotal\to\frac{585162.}{\rho bulk}\right\}$

In[76]:= Vtotal /. % /. ρbulk → ρ0

Out[76]= 0.0585162

We see: The diode is already thick enough for no sup-

ply voltage for this low doping.
At higher doping, we need some voltage, which can be checked by changing $\rho 0$ to $100\rho 0$.