

Solutions to Exercise: Noise RC

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Defaults

```
In[114]:= $Assumptions =  $\omega \in \text{Reals} \ \&\& \ \tau > 0$ ;
```

```
In[115]:= Clear[k, T, R, C];
```

Clear: Symbol C is Protected.

2. Gain (square of transfer function)

```
In[116]:= H[ $\omega$ _] =  $\frac{1}{1 + i \omega \tau}$ ; (* transfer function of a low pass *)
```

```
In[117]:= v2[ $\omega$ _] = H[ $\omega$ ] Conjugate[H[ $\omega$ ]] // Simplify (* gain2*)
```

```
Out[117]=  $\frac{1}{1 + \tau^2 \omega^2}$ 
```

3. Noise Integral

```
In[118]:= noise2 = Integrate[4 k T R v2[2  $\pi$   $\nu$ ], { $\nu$ , 0,  $\infty$ }] /.  $\tau \rightarrow R C$ 
```

```
Out[118]=  $\frac{k T}{C}$ 
```

4. Why independent of R?

Voltage noise INcreases with R, but the bandwidth of the filter DEcreases by the same factor.

5. Brick Wall Equivalent

```
In[119]:= noise2brick = Integrate[4 k T R, { $\nu$ , 0,  $\nu$ brick}] /.  $\tau \rightarrow R C$ 
```

```
Out[119]= 4 k R T  $\nu$ brick
```

```
In[120]:= Solve[noise2 == noise2brick,  $\nu$ brick] // First
```

```
Out[120]=  $\left\{ \nu \text{brick} \rightarrow \frac{1}{4 C R} \right\}$ 
```

6. Comparison to Bandwidth

$$\text{In[121]:= } \frac{2 \pi \nu_{\text{brick}} / . \%}{1 / (R C)}$$

$$\text{Out[121]:= } \frac{\pi}{2}$$