Readout Electronics

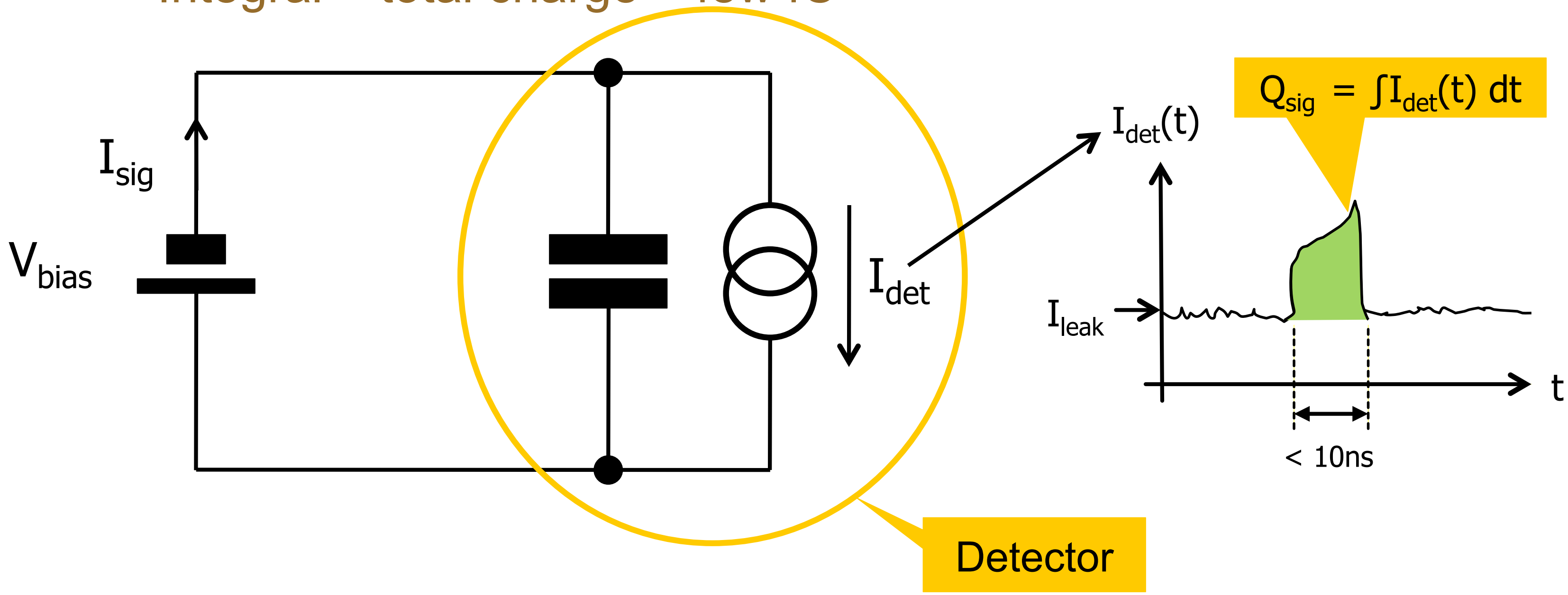
P. Fischer, Heidelberg University

We will treat the following questions:

1. How is the sensor modeled ?
2. What is a typical amplifier arrangement ?
3. What is the output signal?
4. How is noise described and what are the dominant contributions?
5. What is the total noise at the output ?
6. How does noise depend on system parameters and how can it be minimized ?
7. What are typical noise figures ?

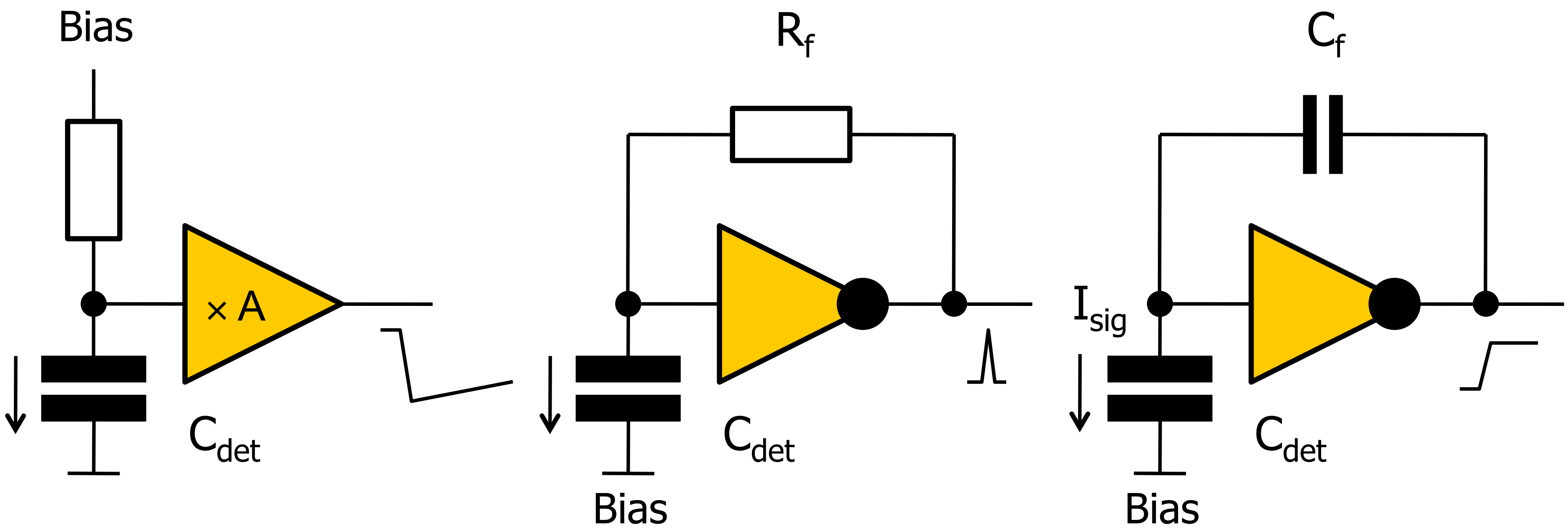
1. How is the sensor modeled ?

- By a capacitance in parallel with a signal current source
  - C: few fF (MAPS, SiDC) – some 100 fF (Pixel) – some 10pF (Strips)
  - I(t): depends on charge motion, O(10ns). Maybe leakage! Integral = total charge = few fC



- Task of the FE Electronics:
  - Fix DC potential of detector on one side
  - Measure signal current / charge

2. What are typical amplifier arrangements ?



Voltage Amplifier

$U_{out} = A\,Q / C_{det}$

- step output
- small for large C_{det}
- need to recharge

Current Amplifier

$U_{out} = I_{sig}\,R_f$

- spike output
- independent of C_{det}

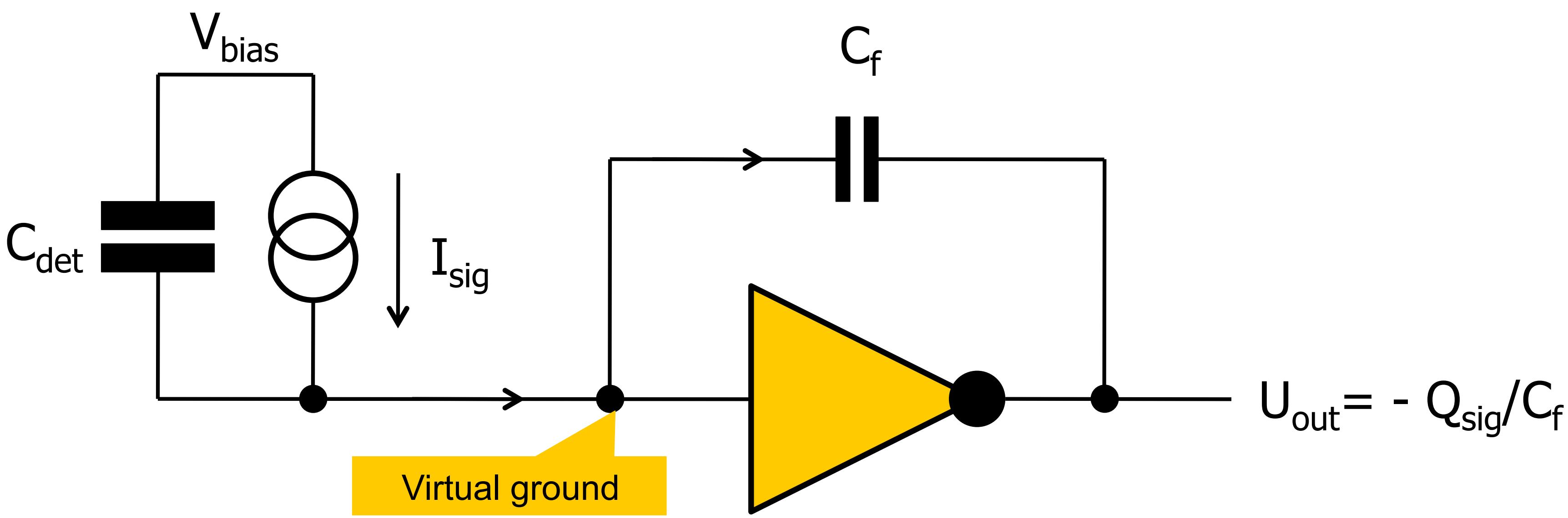
Charge Amplifier

$U_{out} = Q_{sig} / C_f$

- step output
- independent of C_{det}

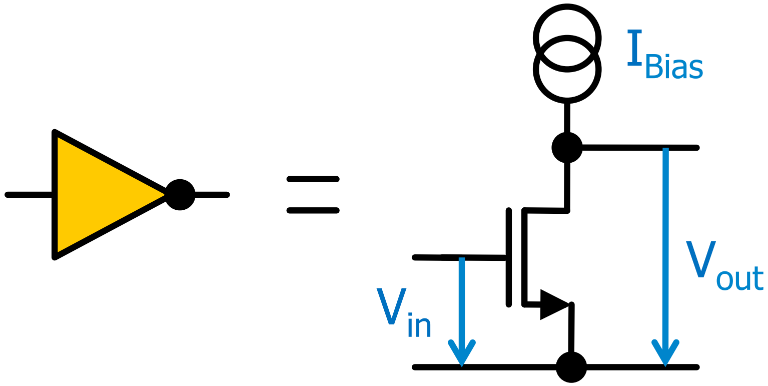
Charge Amplifier in more Detail

- The amplifier generates a *virtual ground* at its input
  - This fixes the potential on the second side of the sensor capacitor (the other side is fixed by V_{bias})
  - Note that in most cases this input voltage is not 0V!
- Current (flowing charge) from the sensor cannot stay on C_{det} (because the voltage is fixed) and must flow onto C_f
  - Therefore $Q_{sig} = \int I_{sig}\, dt = Q_f = U_f\, C_f \quad \rightarrow U_{out} = -U_f = -Q_{sig}/C_f$

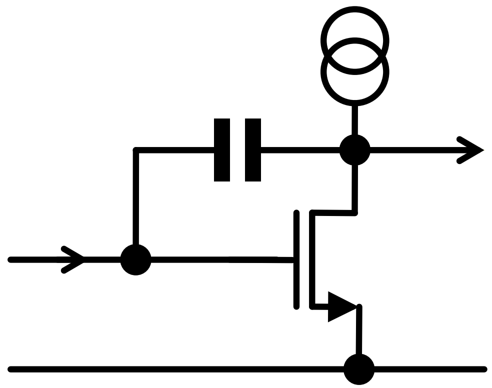


How is the Amplifier Implemented ?

- One transistor can be used as an amplifier:



- A charge amplifier is then very simple:

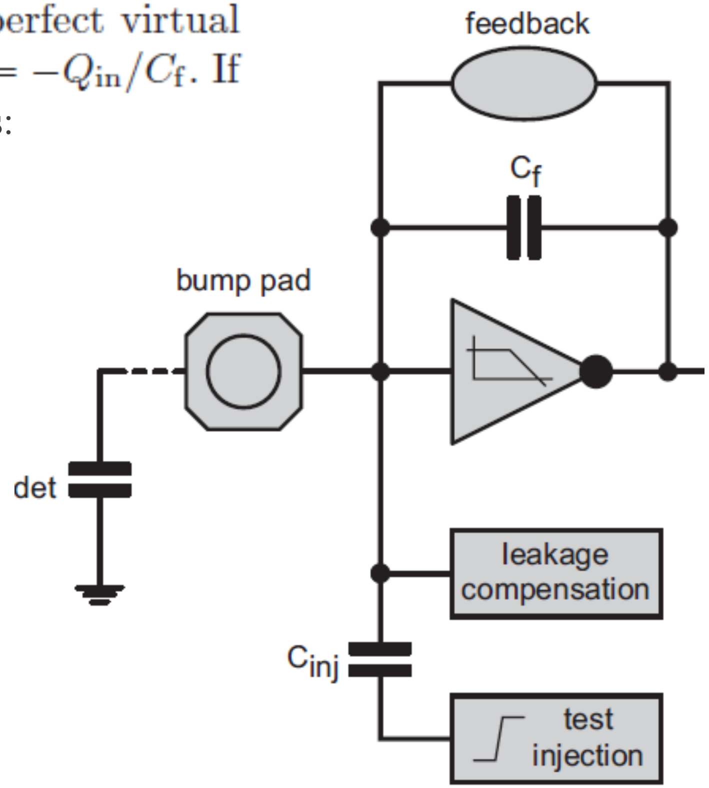


- This simple circuit has (often too) low (voltage) gain.
  A 'Cascode' is often used to increase the gain to >100

Do we get all the Charge?

- What happens if gain of amplifier is finite?

An inverting amplifier with a feedback capacitor C_f converts an input charge Q_{in} to a voltage. An infinite gain would keep the input at a perfect virtual ground and the output voltage step in this ideal case is $\Delta U_{out} = -Q_{in}/C_f$. If the gain $-v_0$ is finite, however, a small residual voltage remains:



$$\Delta U_{in} = \frac{Q_{in}}{C_{in} + (1 + v_0)\, C_f}$$

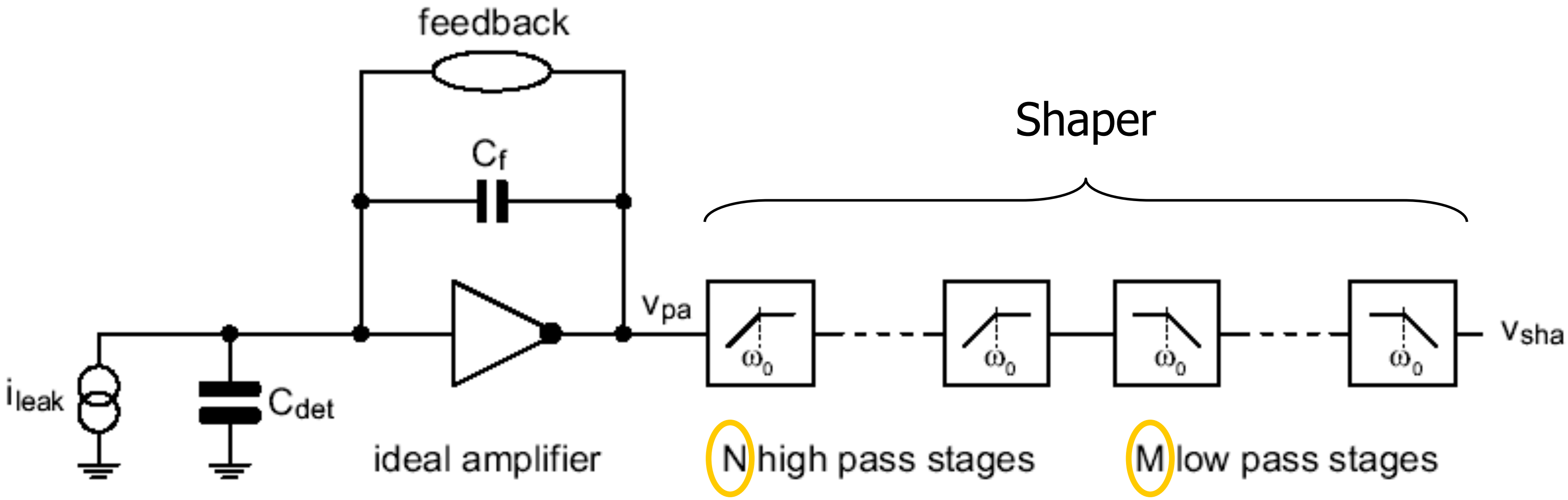
$$C_{eff} = (1 + v_0)\, C_f \approx v_0 C_f$$

$$\Delta U_{out} = -\frac{Q_{in}}{C_f} \cdot \frac{1}{1 + \frac{1}{v_0} + \frac{C_{in}}{v_0 C_f}}$$

$$C_{eff} \approx v_0 C_f \gg C_{in} = C_{det} + C_{amp} + C_{stray}$$

Classical System have a Filter = Shaper

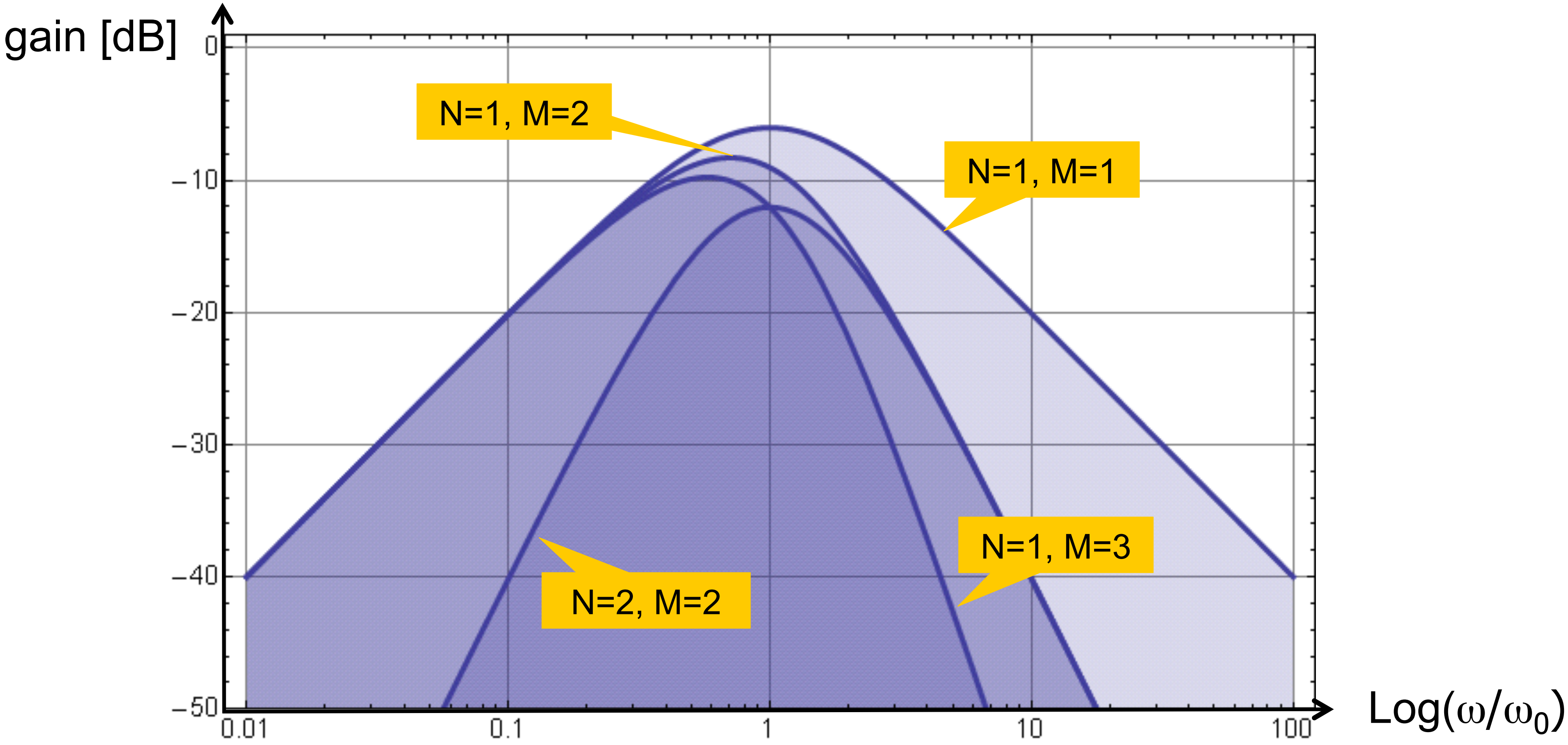
- Filter for pulse shaping & noise reduction:
  - *High pass stages* eliminate DC components & low freq. noise
  - *Low pass stages* limit bandwidth & therefore high freq. noise



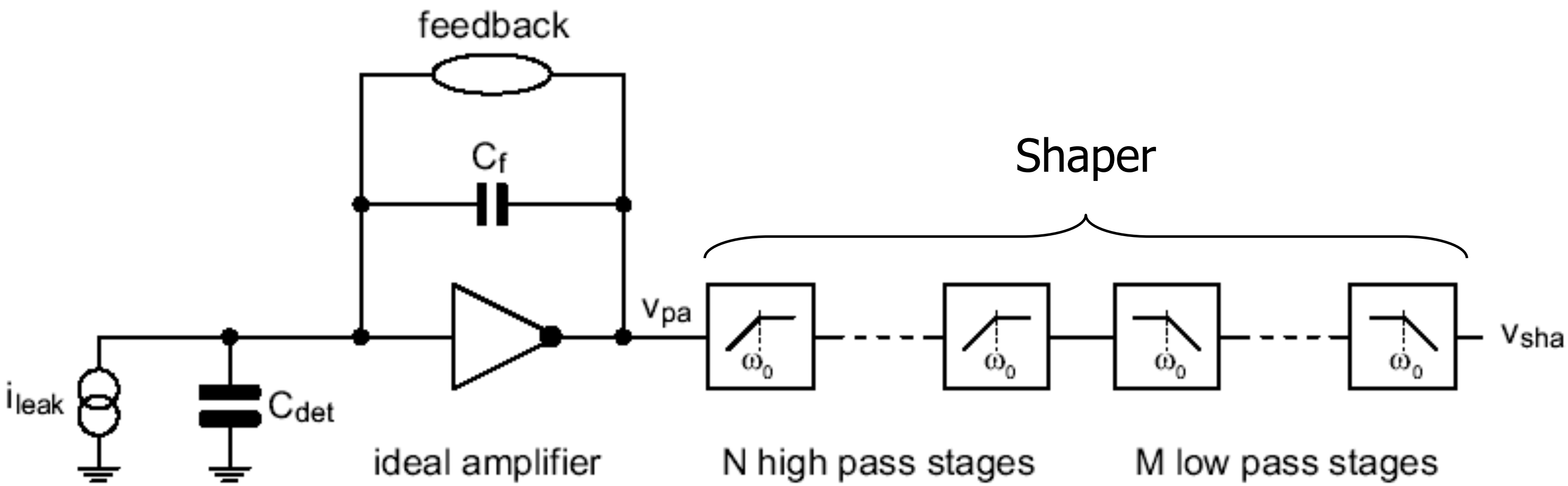
- Due to its output shape (see later), this topology is often called a 'Semi Gaussian Shaper'
- Nearly always N = 1. Often M = 1, sometimes M up to 8

Frequency Behaviour of Shaper

- Low and High frequencies are attenuated
- Corner frequency (here: 1) is transmitted best
- Bode Plot (log/log) of transfer characteristic:



3. What is the output signal?



- For a delta current pulse, the output voltage v_{pa} is a step function
- This has a Laplace-Transform ~1/s
- The transfer functions of the high / low pass stages multiply to:

$$\mathcal{L}^{(N,M)}(s) = \frac{1}{s}\left(\frac{s\tau}{1+s\tau}\right)^N \left(\frac{1}{1+s\tau}\right)^M = \frac{\tau^N s^{N-1}}{(1+s\tau)^{N+M}}$$

Pulse shape after shaper

- The time domain response is the inverse Laplace transform.
- The Laplace integral can be solved with residues:
  There is an (N+M)-fold pole at -1/τ

$$
\begin{aligned}
f^{(N,M)}(t) &= \mathrm{Res} \left. \frac{\tau^N s^{N-1} \mathrm{e}^{st}}{(1+s\tau)^{N+M}} \right|_{s=-1/\tau} \\
&= \frac{\tau^N}{(N+M-1)!} \lim_{s \to -\frac{1}{\tau}} \frac{\mathrm{d}^{N+M-1}}{\mathrm{d}s^{N+M-1}} \left[\frac{s^{N-1} \mathrm{e}^{st}}{(1+s\tau)^{N+M}} \left(s + \frac{1}{\tau} \right)^{N+M} \right] \\
&= \frac{1}{(N+M-1)!} \left(\frac{t}{\tau} \right)^M \sum_{i=0}^{\infty} \frac{(-\frac{t}{\tau})^i}{i!} \frac{(M+i+N-1)!}{(M+i)!} \qquad (1.42)
\end{aligned}
$$

- For only ONE high pass section (N=1), this simplifies to:

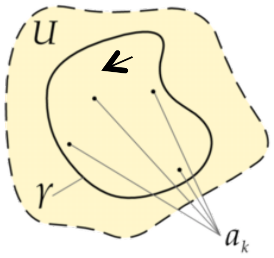
$$
f^{(1,M)}(t) = \frac{1}{M!} \left(\frac{t}{\tau} \right)^M \mathrm{e}^{-t/\tau} \qquad t_{\mathrm{peak}}^{(1,M)} = M\tau = \frac{M}{\omega_0} \qquad f_{\mathrm{max}}^{(1,M)} = \frac{1}{M!} \left(\frac{M}{\mathrm{e}} \right)^M
$$

Reminder (hopefully..): Integration with Residues

This is very simplified!
The statements are valid under certain conditions only.
Consult a book on Complex Analysis!

- The *Residue Theorem* states that the line integral of a function *f* along a closed curve γ in the complex plane is basically the sum of the residues at the *singularities* a_k of *f*:

$$\oint_\gamma f(z)\,dz = 2\pi i \sum \operatorname{Res}(f, a_k)$$



Wikipedia

- The *residue* is a characteristic of a singularity a_k
  - For a first order (simple) pole (f behaves ~ like 1/z at the pole):

$$\operatorname{Res}(f, c) = \lim_{z \to c} (z - c) f(z).$$

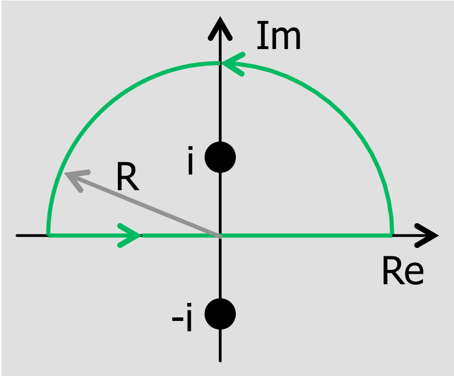
  - For a pole of order n:

$$\operatorname{Res}(f, c) = \frac{1}{(n-1)!} \lim_{z \to c} \frac{d^{n-1}}{dz^{n-1}} \left((z - c)^n f(z) \right).$$

Example for Integration with Residues

- We want to find $A = \int_{-\infty}^{\infty} \frac{1}{x^2+1}$.
- The function $f(z) = \frac{1}{z^2+1} = \frac{1}{(z+i)(z-i)}$ has poles i and $-i$
- The residue at i is:

$$Res(f,i) = \lim_{z \to i} f(z)(z-i) = \lim_{z \to i} \frac{1}{(z+i)} = \frac{1}{2i}$$



- The integral along green curve is then

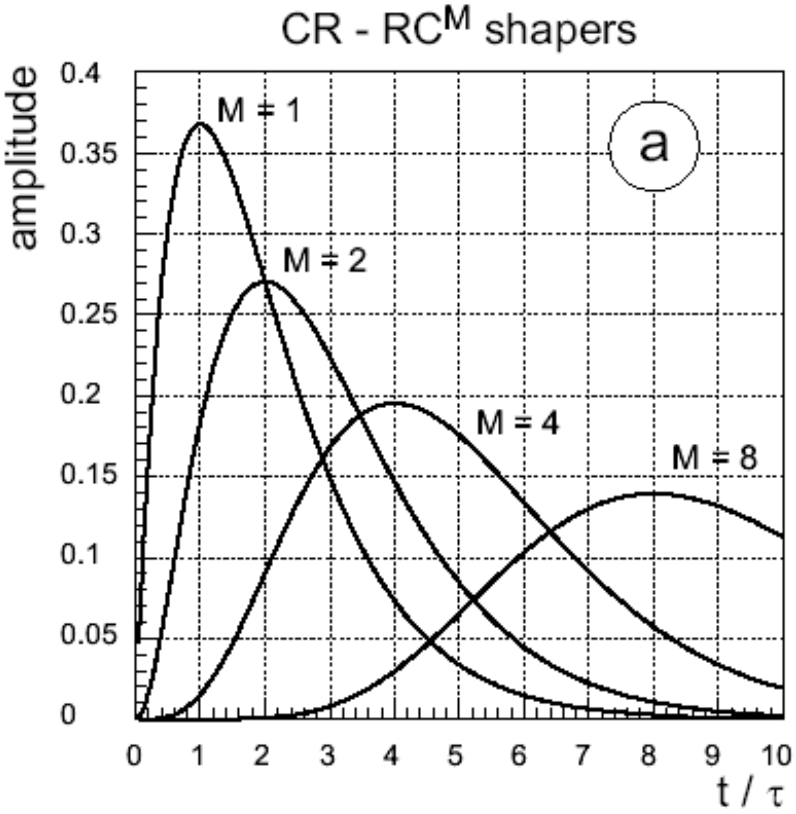
$$\int_C f(z)dz = 2\pi i \, Res(f,i) = \pi$$

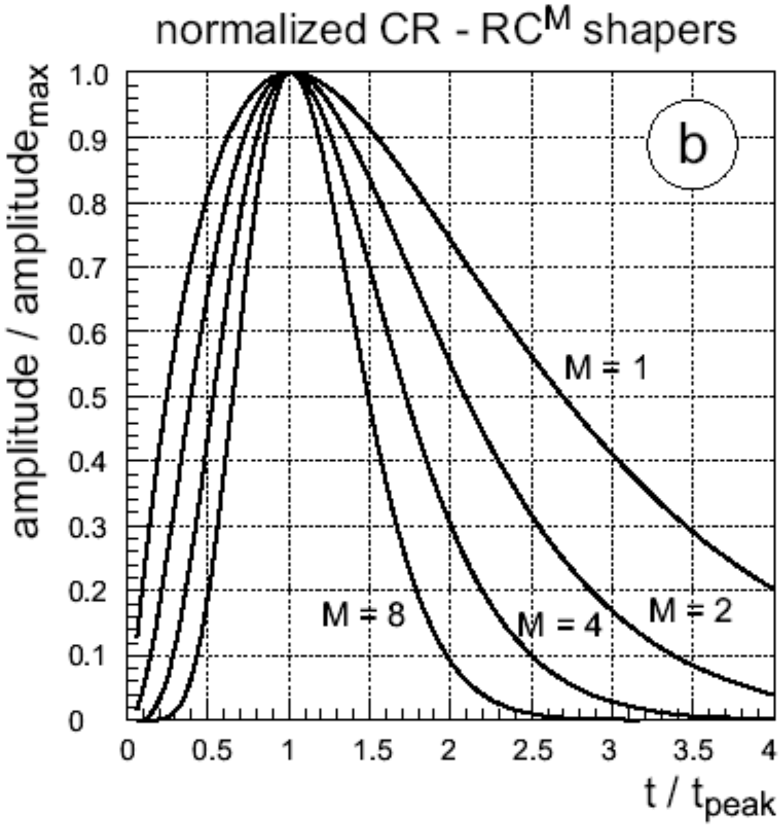
- When we increase the size of the curve, the contribution of the upper arc vanishes* and the lower line becomes A

*: the length of the arc rises ~R, but f falls as $1/R^2$

Pulse Shapes for N=1

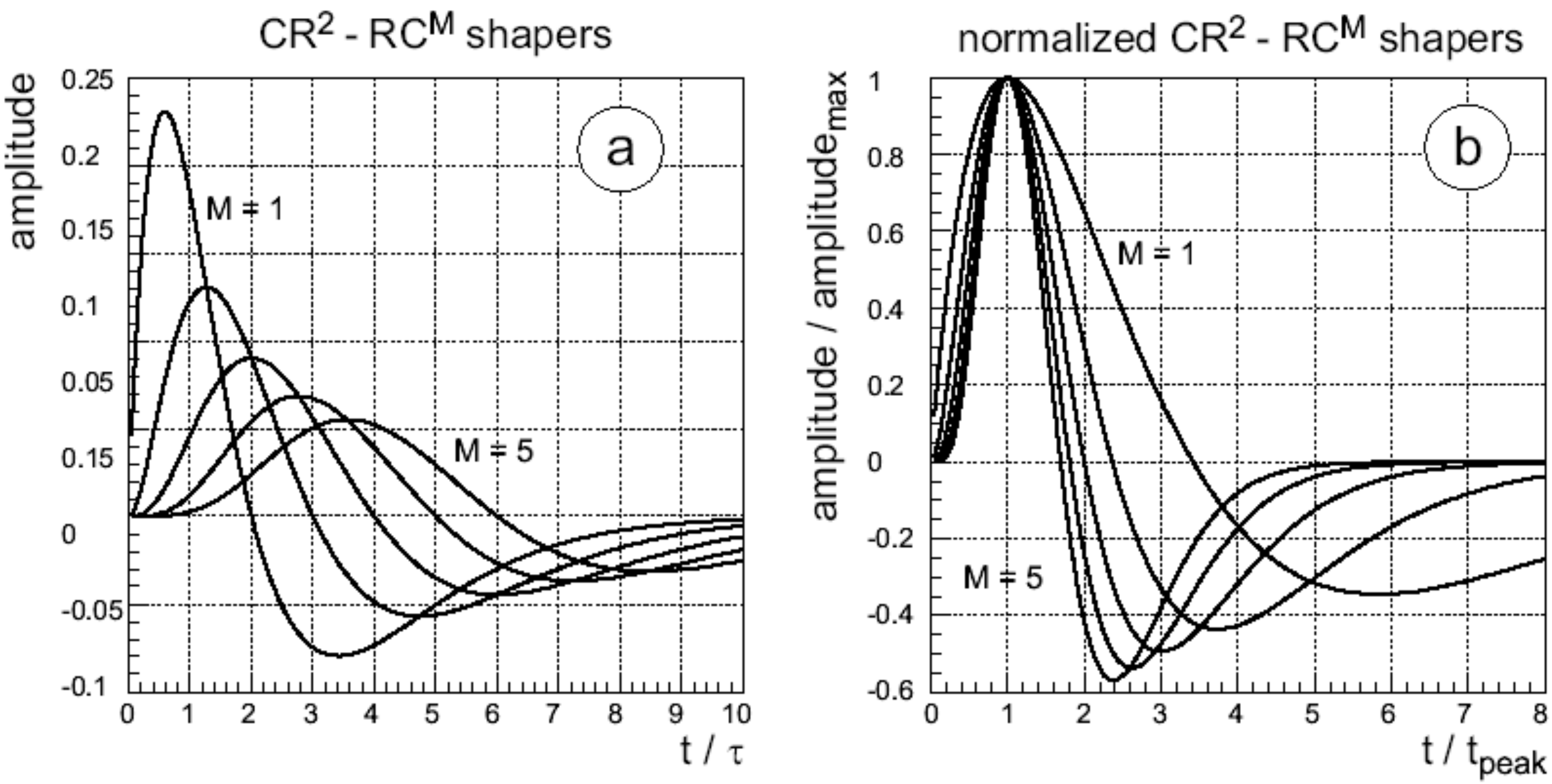
- Pulses from higher order are slower. To keep peeking time, τ of each stage must be decreased
- Right plots shows normalized pulses (same peak amp. & time)
- For high orders, pulses become narrow (width / peaking time), this is good for high pulse rates!



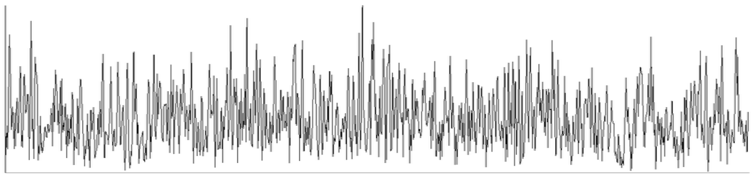


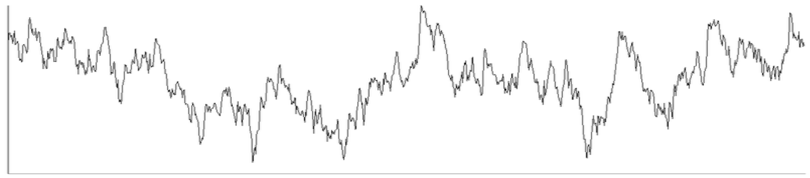
Pulse Shapes for N=2

- This gives an undershoot which is often undesirable → N=1.
  - But: The zero crossing time is *independent* of amplitude.
    It can be used to measure the pulse arrival time with no time walk



4. How is noise described ?





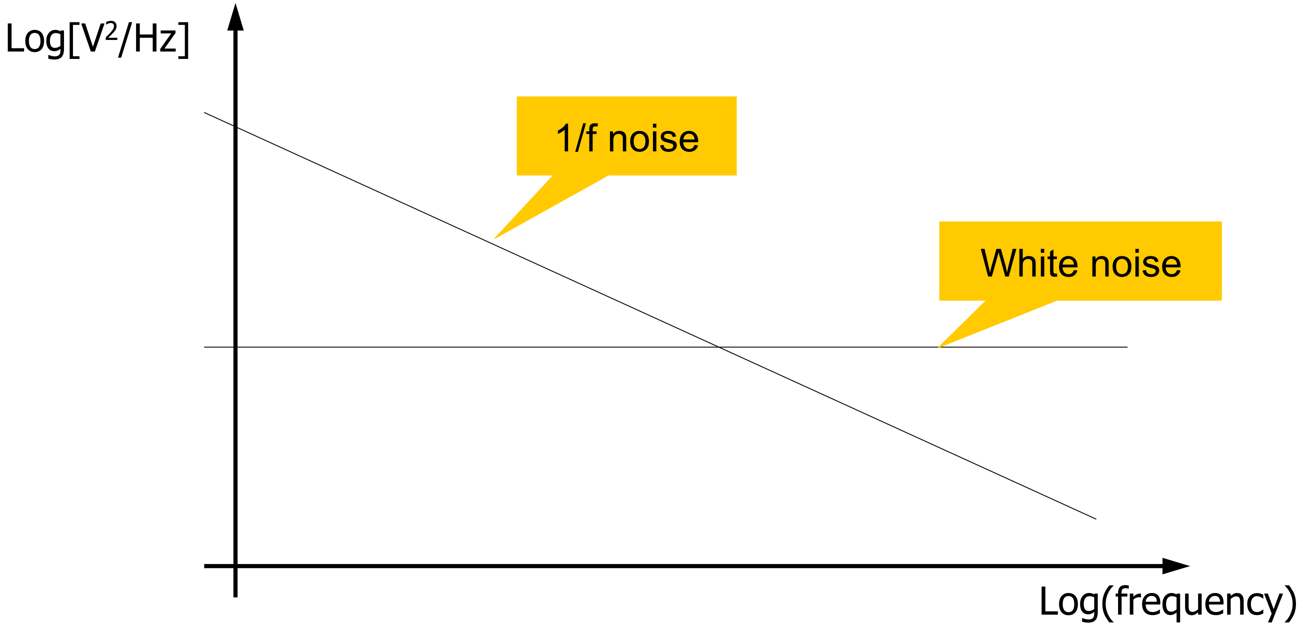
- Noise are random fluctuations of a voltage / current
- The average noise is zero: $\langle sig \rangle = 0$
- The noise 'value' can be defined as the rms: $noise^2 = \langle sig^2 \rangle$

- The fluctuations can have different strength for various frequencies. We therefore describe noise by its *spectral density*, the (squared) noise voltage (density) as a function of frequency. Unit = V^2/Hz (or sometimes $V/\sqrt{Hz}$)

- Spectra can be
  - Constant – 'White Noise'
  - 1/f – '1/f noise'
  - Drop at high freq. – 'pink noise'

Noise Types

- Most common types are
  - *White noise* has constant spectral density
  - The spectral density of *1/f* is ~ 1/f (or $S(f) \propto 1/f^{\alpha}$)
- Be careful: one can use frequency ν, to angular freq. ω!



- The rms noise is the integral of the noise spectral density over all frequencies (0 to ∞)

A Closer Look on Thermal Noise

- Problem: a constant spectral density up to infinite frequencies would be infinite noise power.
- Quantum mechanics gives the exact value for the spectral noise density as a function of frequency ν and temperature T:

$$S_{noise}(\nu, T) = \frac{h\,\nu}{e^{h\nu/kT} - 1} \xrightarrow[h\nu \ll kT]{} kT$$

  - h = Planck's constant = 6.626×10^{-34} Js,
  - k = Boltzmann's constant = 1.381×10^{-23} J/K
- For 'low' frequencies (hν « kT), this is gives just kT
- The noise starts to drop at ν = kT/h ≈ 21 GHz × T/K
  - At room temperature, this is ~ 5THz. The approximation of S_{noise} = kT is therefore valid for all practical circuit frequencies.

- (At very high frequencies, there is an additional 'quantum' noise which rises as hν)

4. What are the important noise sources ?

- The most important noise sources are:
  - *Detector* leakage current (white) (from charge statistics, 'shot noise')

$$\frac{d\langle i^2_{\text{leak}}\rangle}{df} = 2qI_{\text{leak}}$$

  - Noise in *resistors* (white) (from thermal charge motion, 'thermal noise') (mainly in feedback resistor)

$$\frac{d\langle i^2_{\text{Rf}}\rangle}{df} = \frac{4kT}{R_f}$$

or

$$\frac{d\langle v^2_{\text{Rf}}\rangle}{df} = 4kTR_f$$

  - Noise in transistors (white and 1/f)
    - transistor channel behaves like a resistor with a reduction of 2/3 due to channel properties (2/3 in strong inversion … ½ in weal inversion)
      → thermal (current) noise at output equivalent to (voltage) noise at gate:

CHECK

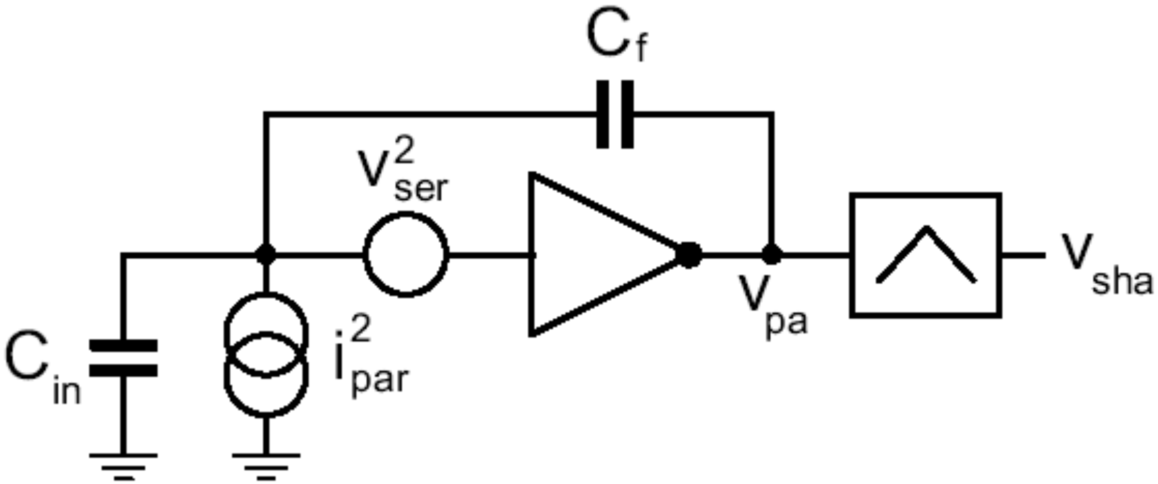
$$\frac{d\langle v^2_{\text{therm}}\rangle}{df} = \frac{8}{3}\frac{kT}{g_m}$$

    - 1/f noise mostly expressed as gate noise voltage:

$$\frac{d\langle v^2_{1/f}\rangle}{df} = \frac{K_f}{C_{ox}WL}\frac{1}{f}$$

Noise calculation: Noise sources

- Equivalent circuit with (ideal) amplifier, input capacitance, feedback capacitance and (dominant) noise sources:



- Spectral densities of noise sources:

$$\text{serial noise voltage}: \quad \frac{\mathrm{d}\langle v^2(f)\rangle}{\mathrm{d}f} = V_0 + V_{-1} f^{-1}$$

$$\text{parallel noise current}: \quad \frac{\mathrm{d}\langle i^2(f)\rangle}{\mathrm{d}f} = I_0$$

white (channel)

1/f noise (MOS)

white (leakage)

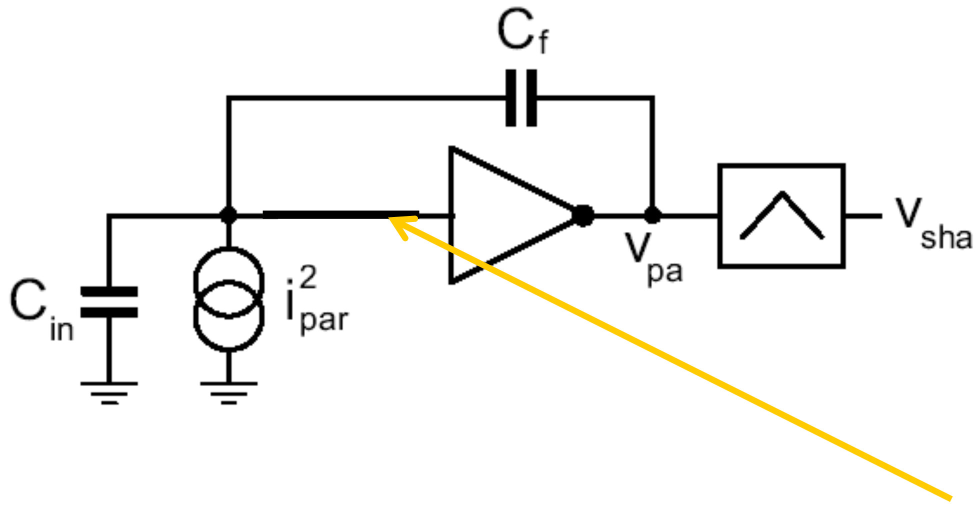
5. What is the total noise at the output ?

- Recipe:
  1. Calculate what effect a voltage / current noise *of a frequency f* at the input has at the output
  2. For each noise source: Integrate over all frequencies (with the respective densities)
  3. Sum contributions of all noise sources

- This yields the total rms voltage noise at the output

- Then compare this to a ‘typical’ signal.
  It is custom to use *one* electron at the input as reference.

Parallel Noise Current



- We assume a perfect virtual ground at the amplifier input

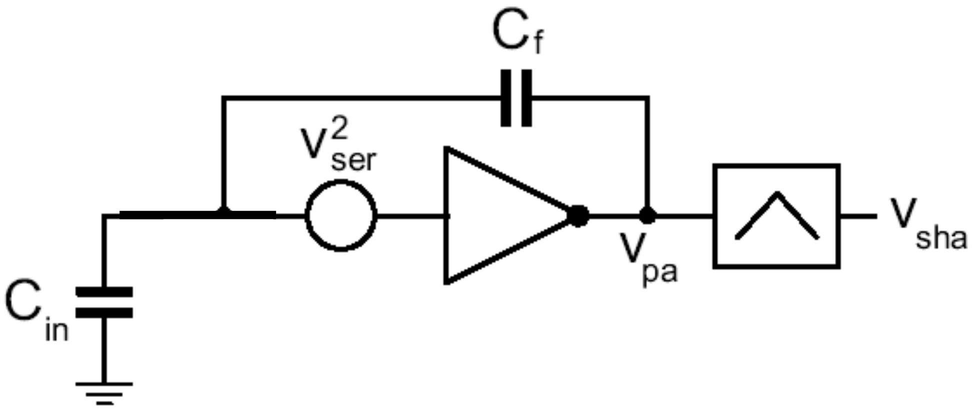
→ No charge can the go to C_{in} (voltages are fixed)

→ Noise current must flow through C_f: $v_{out} = i_{in} \times Z_{Cf}$

$$\text{parallel noise}: \quad \frac{d\langle v_{pa}^2(\omega)\rangle}{d\omega} = \frac{d\langle i_{par}^2(\omega)\rangle}{d\omega} \frac{1}{(\omega C_f)^2} = \frac{I_0}{2\pi} \frac{1}{(\omega C_f)^2}$$

(note the change of the frequency variable from ν to ω)

Serial Noise Voltage



- Output noise is determined by the capacitive divider made from C_f and C_{in}: $v_{ser} = v_{pa} \times Z_{Cin} / (Z_{Cin}+Z_{Cf})$

or: $$v_{\rm pa}^2 = v_{\rm ser}^2 \left(\frac{C_{\rm in}+C_{\rm f}}{C_{\rm f}}\right)^2$$

Therefore: $$\text{serial noise}: \quad \frac{{\rm d}\langle v_{\rm pa}^2(\omega)\rangle}{{\rm d}\omega} \approx \left(V_{-1}\omega^{-1} + \frac{V_0}{2\pi}\right)\left(\frac{C_{\rm in}}{C_{\rm f}}\right)^2$$

($C_{in}=C_{det} + C_{preamp} + C_{parasitic}$)

Total *Output* Noise (after the amplifier)

- In total, the output noise can be written as a sum of contributions with different frequency dependence:

$$\frac{\mathrm{d}\langle v_{\mathrm{pa}}^2(\omega)\rangle}{\mathrm{d}\omega} = \sum_{k=-2}^{0} c_k \omega^k$$

frequency dependence is here

with

$$c_{-2} = \frac{I_0}{2\pi C_\mathrm{f}^2}, \quad c_{-1} = V_{-1}\frac{C_\mathrm{in}^2}{C_\mathrm{f}^2} \quad \text{and} \quad c_0 = V_0 \frac{C_\mathrm{in}^2}{2\pi C_\mathrm{f}^2}.$$

leakage (white) | MOS gate (1/f) | MOS channel (white)

from leakage current I_leak: $\quad I_0 = 2qI_\mathrm{leak}$

from transistor channel noise: $\quad V_0 = \frac{8}{3}\frac{kT}{g_m}$

from 1/f noise: $\quad V_{-1} = \frac{K_\mathrm{f}}{C_\mathrm{ox}WL}$

Noise Transfer Function

- (N,M) - Shaper transfer function: $H^2_{N,M}(\omega) = A^2 \frac{(\frac{\omega}{\omega_0})^{2N}}{\left(1+(\frac{\omega}{\omega_0})^2\right)^{N+M}}.$

- 'Filtered' noise at the output of the shaper:

$$\langle v^2_{\mathrm{sha}}\rangle = \int_0^\infty H^2_{N,M}(\omega)\,\mathrm{d}\langle v^2_{\mathrm{pa}}(\omega)\rangle = \sum_{k=-2}^{0}\int_0^\infty c_k\,\omega^k H^2_{N,M}(\omega)\mathrm{d}\omega$$

$$= \frac{A^2}{2}\frac{1}{\Gamma(N+M)}\sum_{k=-2}^{0} c_k\,\omega_0^{k+1}\,\Gamma\left(N+\frac{k+1}{2}\right)\Gamma\left(M-\frac{k+1}{2}\right)$$

$$\Gamma(z+1) = z\,\Gamma(z)$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi},$$

- For simplest shaper (N=M=1),
  Squared rms noise voltage at the shaper output:

$$\langle v^2_{\mathrm{sha}}\rangle = A^2\frac{\pi}{4}\left(\frac{c_{-2}}{\omega_0} + \frac{2}{\pi}c_{-1} + \omega_0\,c_0\right)$$

Calculation of ENC

- The **equivalent noise charge, ENC** is the (rms) noise at the output of the shaper expressed in Electrons input charge, i.e. divided by the 'charge gain'
- The 'charge gain' is (see before): Vmax = q/C_f × A × 1/e

charge of 1 electron (1.6e-19C)

Shaper dc gain

Peak amplitude for N=M=1

leakage gives noise for slow shaping

1/f – noise cannot be reduced by charging shaping time

$$\mathrm{ENC}^2_{\mathrm{CR-RC}} = \frac{\langle v^2_{\mathrm{sha}} \rangle}{V^2_{\mathrm{max}}} = \frac{\mathrm{e}^2}{4q^2}\left(\frac{\tau}{2} I_0 + \frac{1}{2\tau} V_0 C^2_{\mathrm{in}} + 2\, V_{-1} C^2_{\mathrm{in}}\right)$$

C_{in} is bad for **fast** shaping.
Reducing V_0 requires large g_m

Noise contributions

- Real noise contributions for the coefficients I_0, V_0, V_{-1}:

from leakage current I_{leak}: $$I_0 = 2qI_{\text{leak}}$$

from transistor channel noise: $$V_0 = \frac{8}{3}\frac{kT}{g_m}$$

from 1/f noise: $$V_{-1} = \frac{K_f}{C_{\text{ox}}WL}$$

- For a 0.25µm technology (C_{ox}=6.4 fF/µm^2, K_f=33×10^{-25} J, L=0.5µm, W=20µm) and C_{in}=200fF, I_{leak}=1nA and τ=50ns, g_m=500µS (typical LHC pixel detector):

$$\left(\frac{\text{ENC}}{\text{e}^-}\right)^2 = 115 \cdot \frac{\tau}{10\,\text{ns}} \cdot \frac{I_{\text{leak}}}{1\,\text{nA}} \quad \rightarrow 575$$

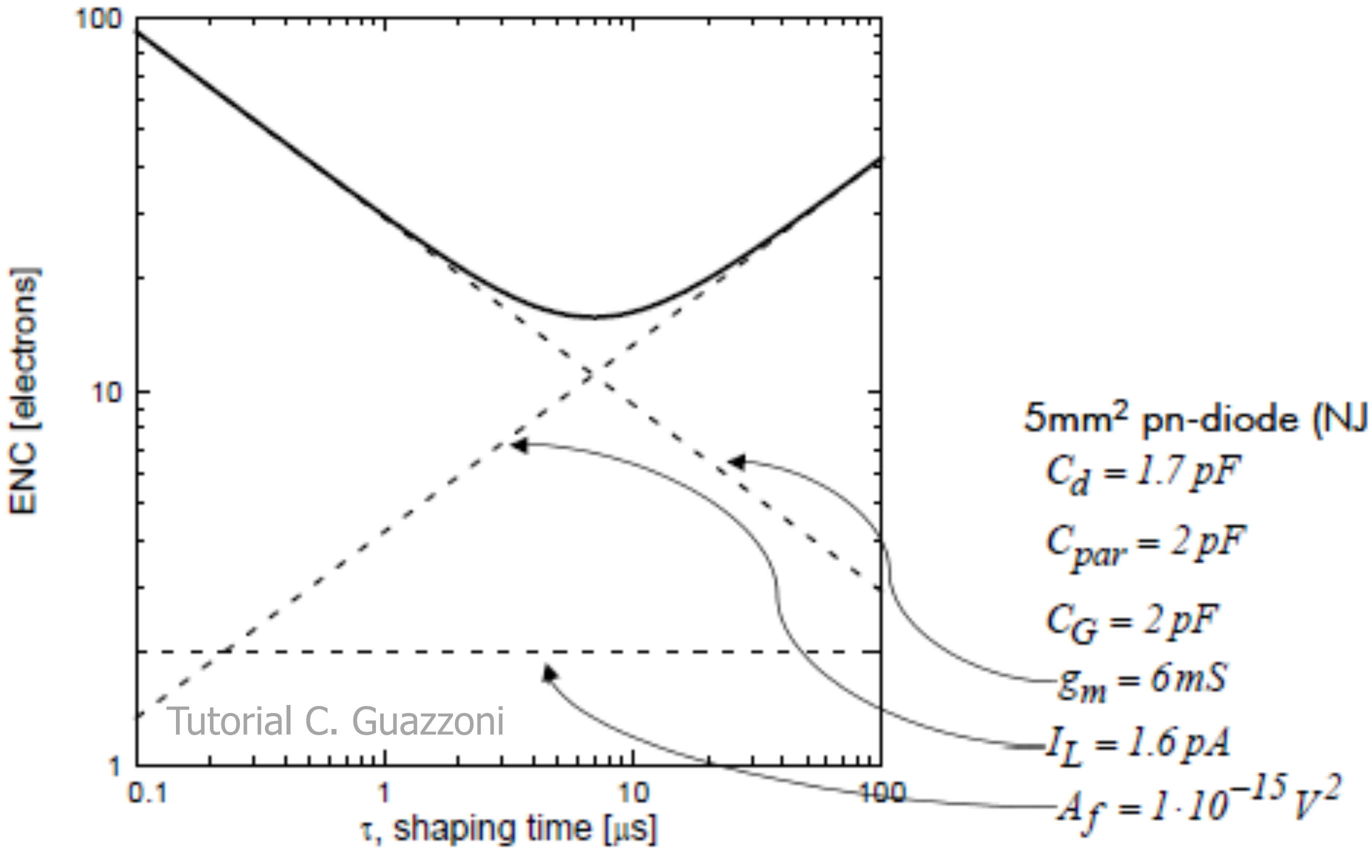
$$+ 388 \cdot \frac{10\,\text{ns}}{\tau} \cdot \frac{\text{mS}}{g_m} \cdot \left(\frac{C_{\text{in}}}{100\,\text{fF}}\right)^2 \quad \rightarrow 621$$

$$+ 74 \cdot \left(\frac{C_{\text{in}}}{100\,\text{fF}}\right)^2 . \quad \rightarrow 296$$

ENC=40 e$^-$

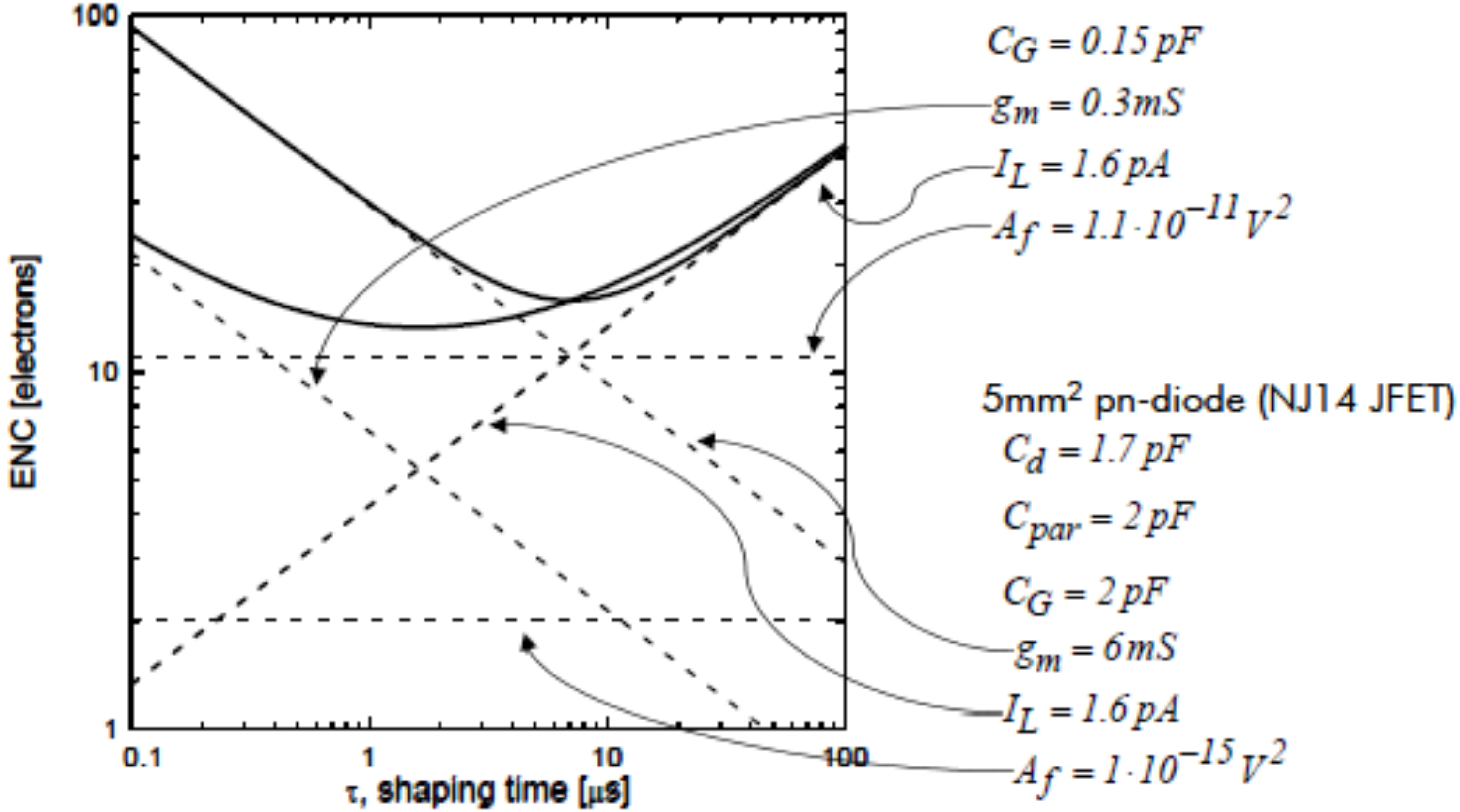
Noise vs. Shaping Time

- Long shaping: leakage noise contributes more
- Short shaping: Amplifier white noise, worsened by C_{Det}
- Always: Amplifier 1/f noise, worsened by C_{Det}



Comparison of two Detector Systems

✓ ENC vs. shaping time (τ)



Typical Noise Values

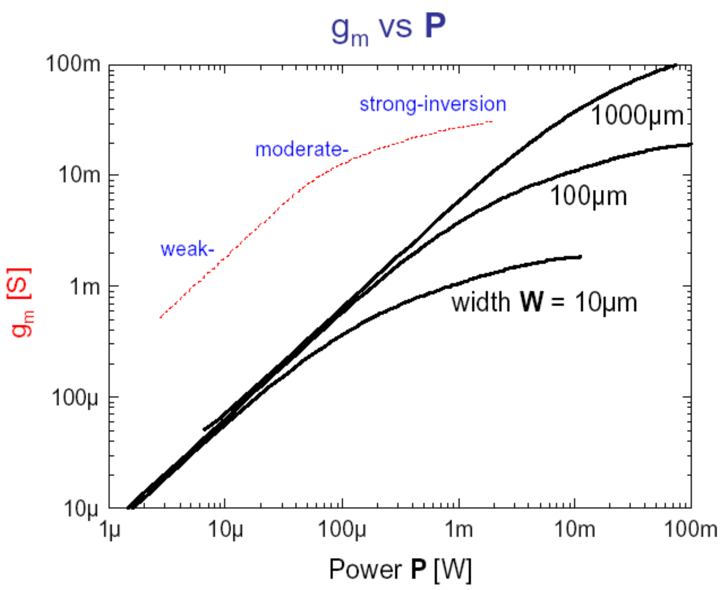
C_{in}	Shaping	Power	Noise	System
10fF	µs	100uW	5	CCD, DEPFET
100fF	µs	40µW	30	Slow Pixel
100fF	25ns	40µW	100	Pixel (ATLAS)
20pF	200ns	1000µW	1000	Strips

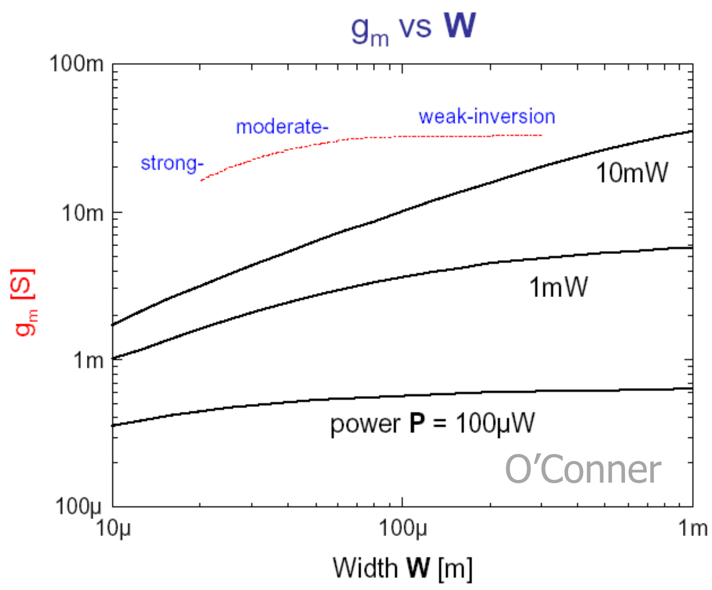
Why Do we Need Low Noise ?

- Spectral resolution
- Position resolution (good interpolation, only for ‘wide’ signals)
- Low noise hit rate (with threshold)
- Good efficiency (with threshold)

How to get g_m? More power and larger W!

- The transconductance g_m of the input MOS is most important. It can be increased by
  - shorter length L (technology limit! short L can add noise)
  - Wider width W works, but increases input capacitance!
  - Increase current works, but increases power consumption





Optimal ENC (Optimal W for each Power value)

Input MOSFET optimization