The pn-Diode

A few Natural Constants

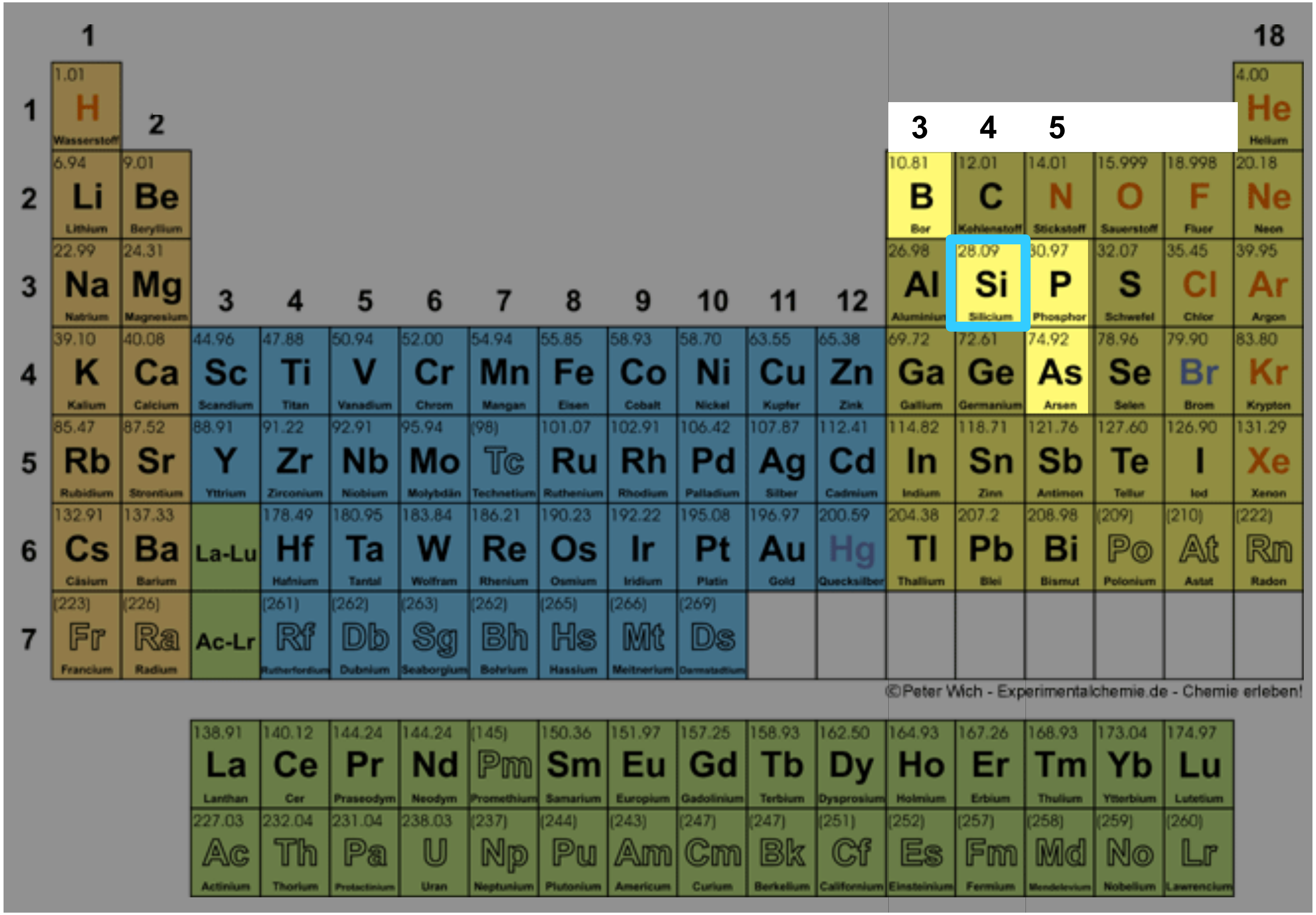
q	1.602×10^{-19} C	elementary charge
k	1.381×10^{-23} J/K	Boltzmann constant
ε_0	8.854×10^{-12} F/m	vacuum susceptibility (Hint: $C = \varepsilon_0 A/d$, 1m x 1m x 1m: ~10pF)
$4kT$	1.657×10^{-20} J	Noise Power density @ 300K
U_T	$= kT/q = 25.9$ mV	Thermal voltage @ 300K

A few Constants for Silicon

E_g	1.12	eV	band gap at 300K
N_{atom}	5×10^{22}	cm^{-3}	atom density
N_i	1.01×10^{10}	cm^{-3}	intrinsic carrier density at 300K[*] (‘old’ value: 1.45)
μ_e	~1400	cm^2/Vs	electron mobility (@ low fields)
μ_h	~480	cm^2/Vs	hole mobility ($v = \mu E$)
E_{cit}	~1	V/µm	critical field where mobility starts to drop
ε_{Si}	11.9		dielectric constant of silicon
ε_{SiO2}	3.90		dielectric constant of silicon - dioxide
E_{max}	$\sim 3 \times 10^7$	V/m	break through field strength
E_{eh}	3.6	eV	Av. Energy required to generate an e-h pair
ρ	7.87	gcm^{-3}	density
λ	150	W / (mK)	thermal conductivity
α	2.56	$10^{-6}\ K^{-1}$	thermal expansion coefficient (compare Al: 23.1)

[*] Sproul AB, Green MA. Improved value for the silicon intrinsic carrier concentration from 275 to 375 K. Journal of Applied Physics. 1991;70:846-854. Available from: http://link.aip.org/link/?JAP/70/846/1

Silicon



	1	2	3	4	5	6	7	8	9	10	11	12	3	4	5			18
1	1.01 H Wasserstoff																	4.00 He Helium
2	6.94 Li Lithium	9.01 Be Beryllium											10.81 B Bor	12.01 C Kohlenstoff	14.01 N Stickstoff	15.999 O Sauerstoff	18.998 F Fluor	20.18 Ne Neon
3	22.99 Na Natrium	24.31 Mg Magnesium											26.98 Al Aluminium	28.09 Si Silicium	30.97 P Phosphor	32.07 S Schwefel	35.45 Cl Chlor	39.95 Ar Argon
4	39.10 K Kalium	40.08 Ca Calcium	44.96 Sc Scandium	47.88 Ti Titan	50.94 V Vanadium	52.00 Cr Chrom	54.94 Mn Mangan	55.85 Fe Eisen	58.93 Co Cobalt	58.70 Ni Nickel	63.55 Cu Kupfer	65.38 Zn Zink	69.72 Ga Gallium	72.61 Ge Germanium	74.92 As Arsen	78.96 Se Selen	79.90 Br Brom	83.80 Kr Krypton
5	85.47 Rb Rubidium	87.52 Sr Strontium	88.91 Y Yttrium	91.22 Zr Zirconium	92.91 Nb Niobium	95.94 Mo Molybdän	(98) Tc Technetium	101.07 Ru Ruthenium	102.91 Rh Rhodium	106.42 Pd Palladium	107.87 Ag Silber	112.41 Cd Cadmium	114.82 In Indium	118.71 Sn Zinn	121.76 Sb Antimon	127.60 Te Tellur	126.90 I Iod	131.29 Xe Xenon
6	132.91 Cs Cäsium	137.33 Ba Barium	La-Lu	178.49 Hf Hafnium	180.95 Ta Tantal	183.84 W Wolfram	186.21 Re Rhenium	190.23 Os Osmium	192.22 Ir Iridium	195.08 Pt Platin	196.97 Au Gold	200.59 Hg Quecksilber	204.38 Tl Thallium	207.2 Pb Blei	208.98 Bi Bismut	(209) Po Polonium	(210) At Astat	(222) Rn Radon
7	(223) Fr Francium	(226) Ra Radium	Ac-Lr	(261) Rf Rutherfordium	(262) Db Dubnium	(263) Sg Seaborgium	(262) Bh Bohrium	(265) Hs Hassium	(266) Mt Meitnerium	(269) Ds Darmstadtium								

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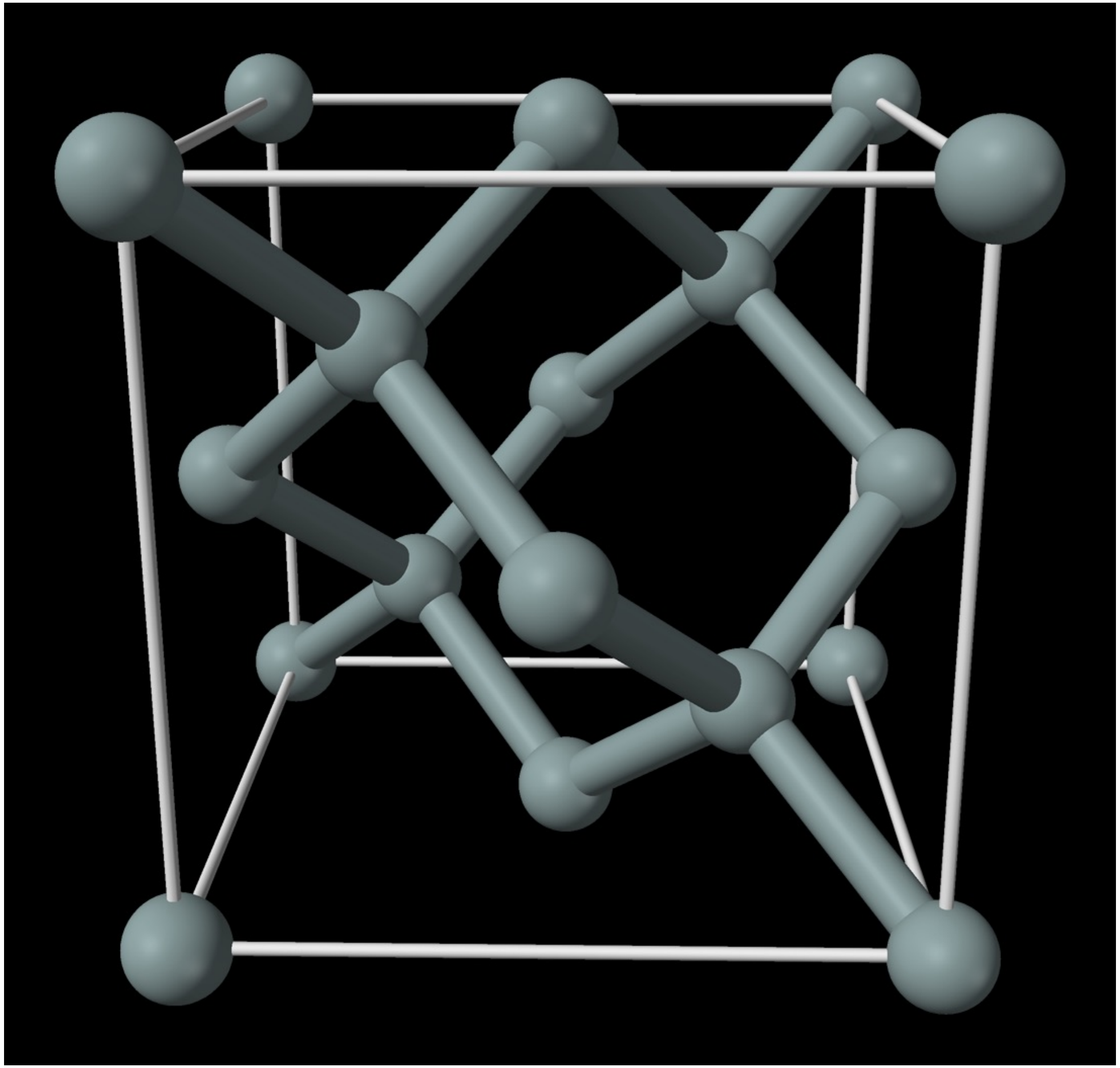
138.91 La Lanthan	140.12 Ce Cer	144.24 Pr Praseodym	144.24 Nd Neodym	(145) Pm Promethium	150.36 Sm Samarium	151.97 Eu Europium	157.25 Gd Gadolinium	158.93 Tb Terbium	162.50 Dy Dysprosium	164.93 Ho Holmium	167.26 Er Erbium	168.93 Tm Thulium	173.04 Yb Ytterbium	174.97 Lu Lutetium
227.03 Ac Actinium	232.04 Th Thorium	231.04 Pa Protactinium	238.03 U Uran	(237) Np Neptunium	(244) Pu Plutonium	(243) Am Americium	(247) Cm Curium	(247) Bk Berkelium	(251) Cf Californium	(252) Es Einsteinium	(257) Fm Fermium	(258) Md Mendelevium	(259) No Nobelium	(260) Lr Lawrencium

Silicon Crystal

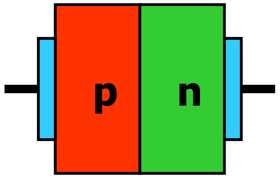


Each atom has
4 bindings.

See VRML File
Diamond.wrl

Face centered
Cubic lattice

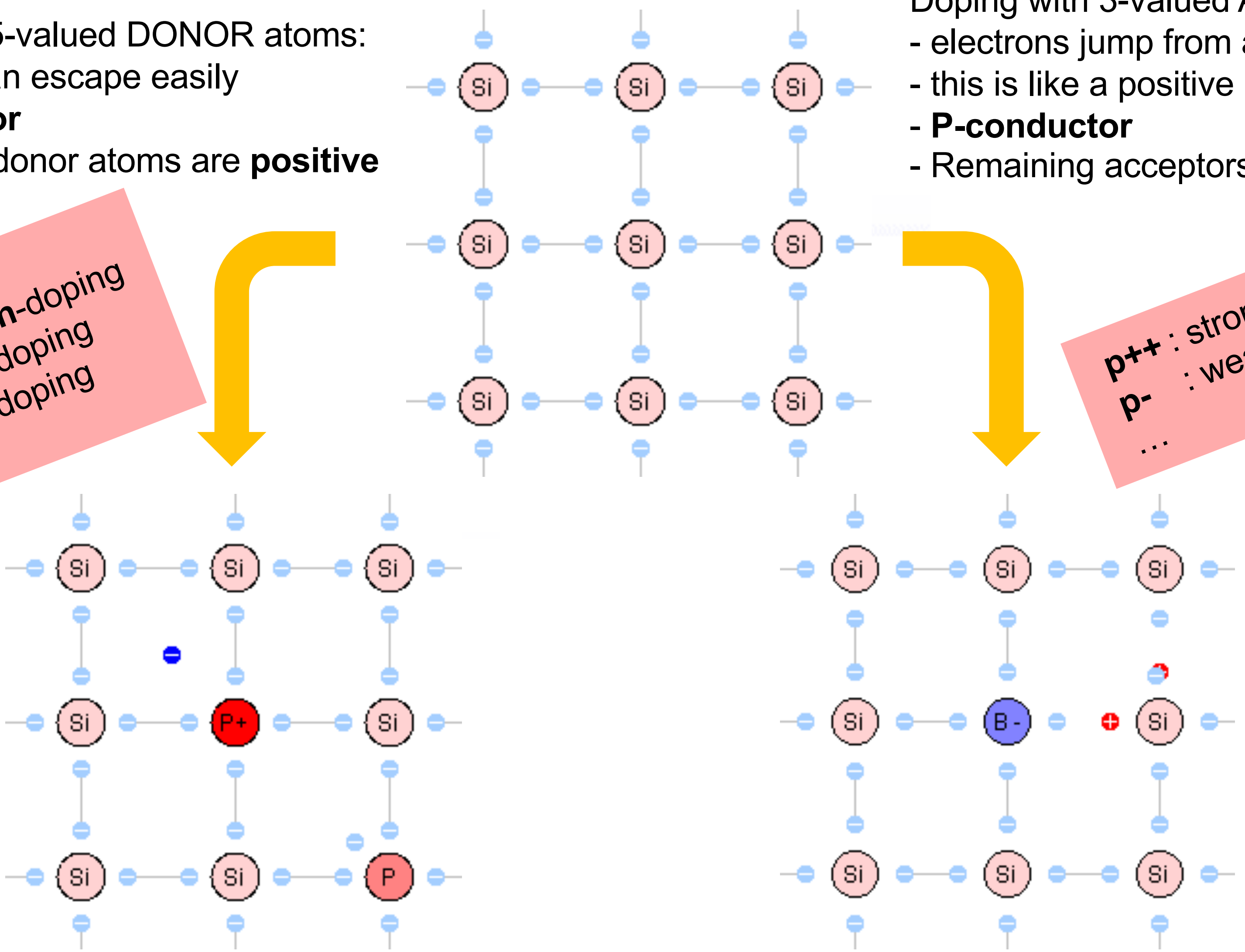
The Diode (p-n-junction)



Silicon: Crystal & Doping

Doping with 5-valued DONOR atoms:
- electrons can escape easily
- **N-conductor**
- Remaining donor atoms are **positive**

Doping with 3-valued ACCEPTORs:
- electrons jump from atom to atom
- this is like a positive moving charge
- **P-conductor**
- Remaining acceptors are **negative**

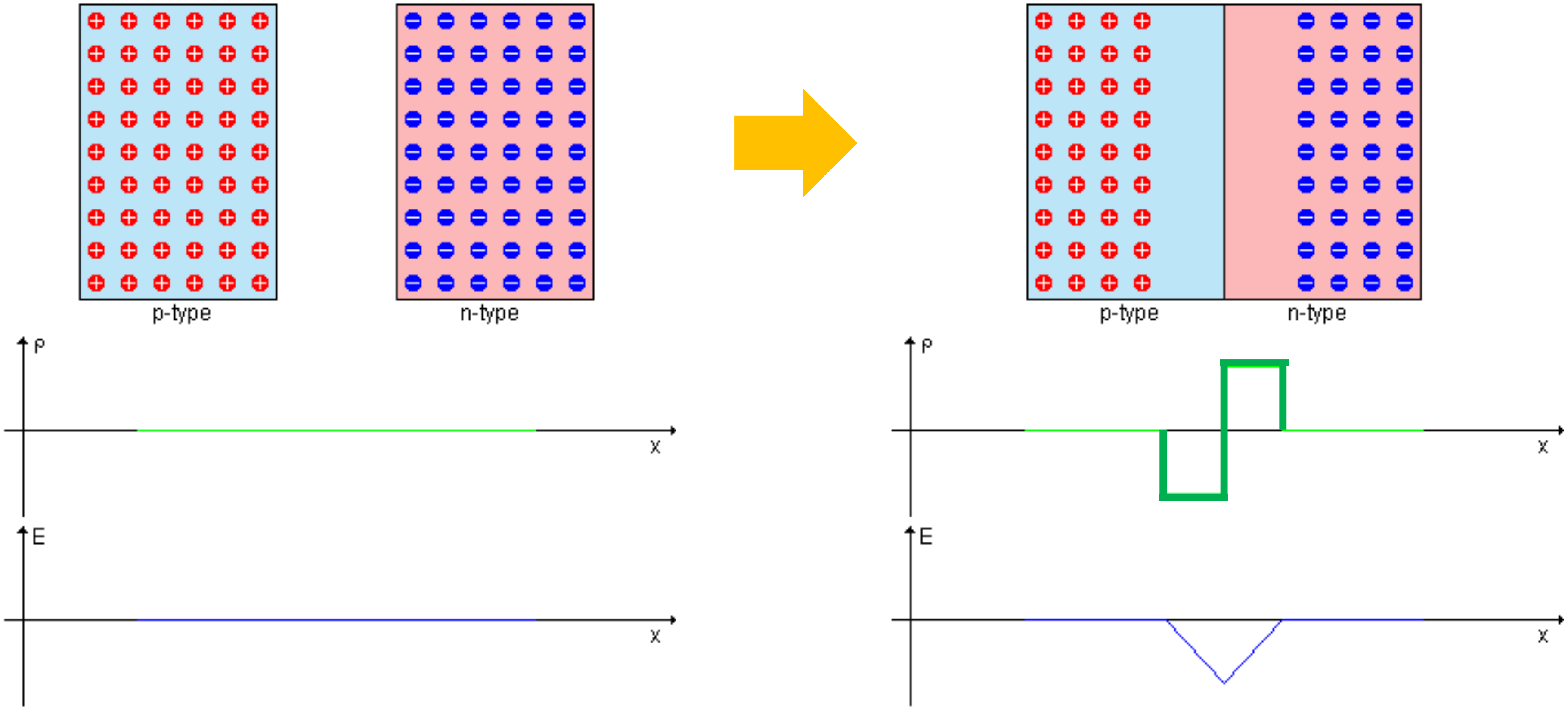


Nomenclature:
n+ : Moderate **n**-doping
n++ : strong **n**-doping
n- : weak **n**-doping
…

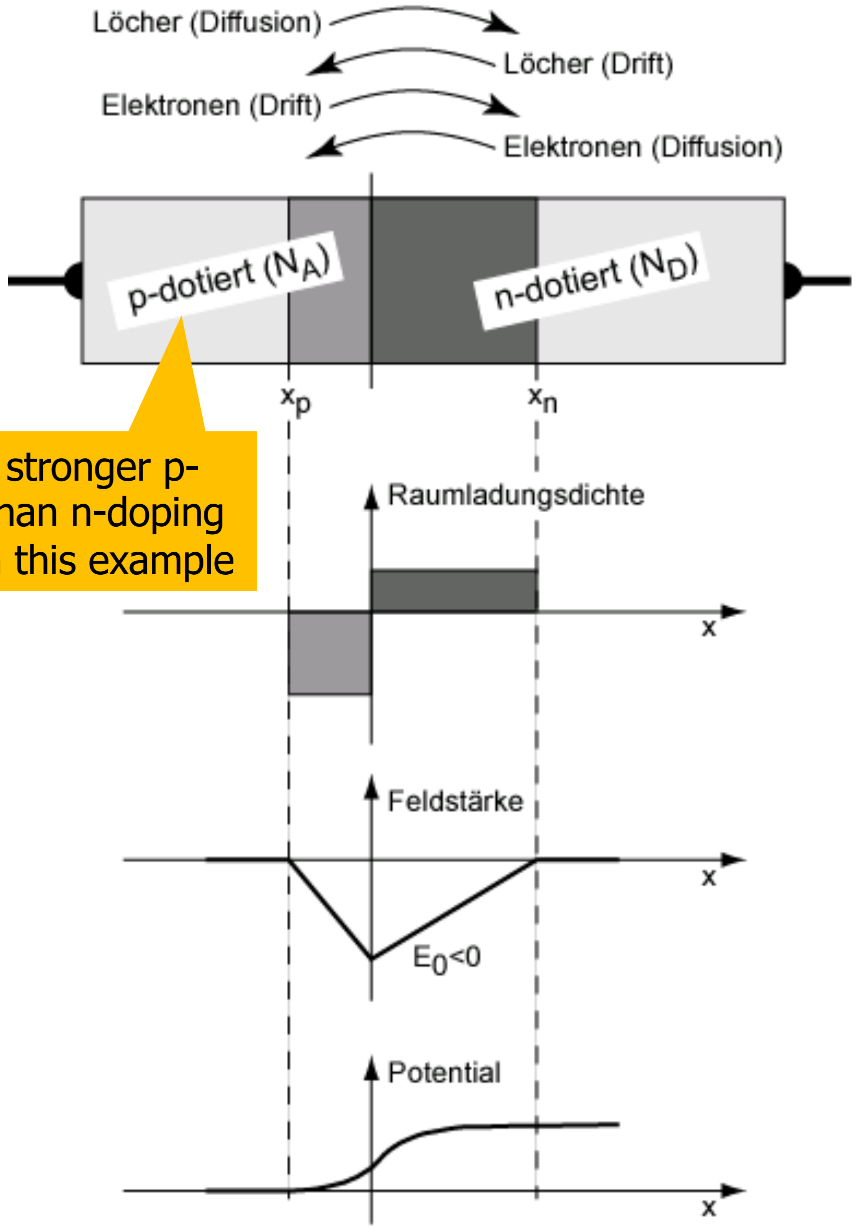
p++ : strong **p**-doping
p- : weak **p**-doping
…

The pn-junction (diode)

- Bringing together a p- and n doped region:
  - A depletion zone with no charge carriers is created
  - There is a space charge
  - -> There is an electric field



Origin of Depletion Layers



stronger p- than n-doping in this example

- We consider an idealized, 'abrupt' transition between n- and p- region (this is smooth in reality)

- Due to the concentration gradient, electrons diffuse from the n → p region (holes from p → n).
- The carriers compensate and we get depleted regions without mobile carriers

- The fixed, ionized atoms are positively charged in the n-region (negatively in the p-region)
- This leads to an electric field
- The field is associated with a electrostatic potential. This ‘built in' potential depends only on doping.
- The field leads to a drift of electrons/holes backwards.
- **The thickness of the depletion region is determined by the equilibrium between drift- and diffusion currents**

- In reality, the depletion zone drops more slowly to zero, but the transition region is small.

Derivation of the Build-In Voltage

- Derivation steps (see extra file on web site for explanations)
  - (p(x)= hole density):

$$
\begin{aligned}
j_{Feld}(x) &= -j_{Diff}(x) \\
q\,\mu\,p(x)E(x) &= q\,D\frac{dp(x)}{dx} \\
-\frac{q}{kT}dV(x) &= \frac{dp(x)}{p(x)} \\
-\frac{q}{kT}\int_{V_p}^{V_n} dV(x) &= \int_{p_p}^{p_n}\frac{dp(x)}{p(x)} \\
-\frac{q}{kT}(V_n - V_p) &= \ln\left(\frac{p_n}{p_p}\right) \\
V_{bi} := V_n - V_p &= \frac{kT}{q}\ln\left(\frac{p_p}{p_n}\right)
\end{aligned}
$$

$$
\begin{aligned}
V_{bi} &= \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) \\
&\approx \left[\log\left(\frac{N_A}{n_i}\right) + \log\left(\frac{N_D}{n_i}\right)\right] \times 60\,\mathrm{mV}
\end{aligned}
$$

V_{bi} is often also called 'Diffusion Voltage'

- kT/q is a quantity which occurs often
- It is called the 'Thermal Voltage'
- It is ~26mV at room temperature

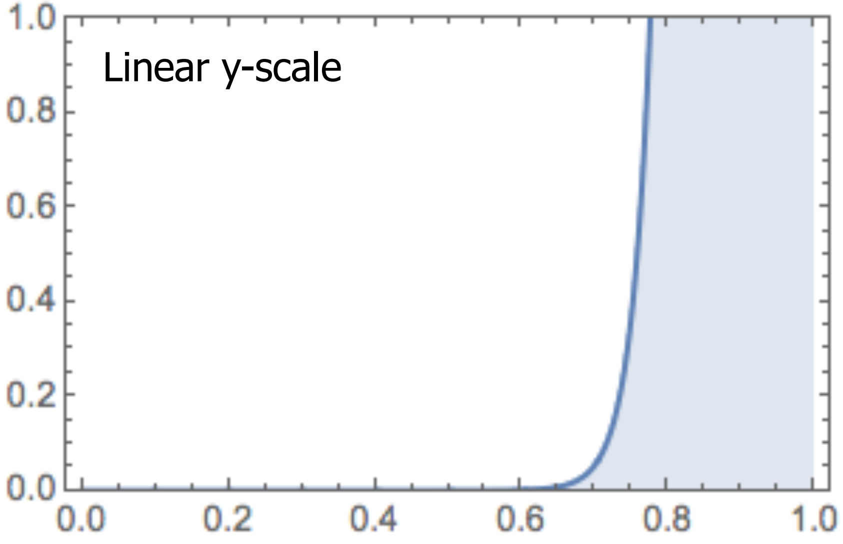
For typical doping concentrations, this is a few 100 mV

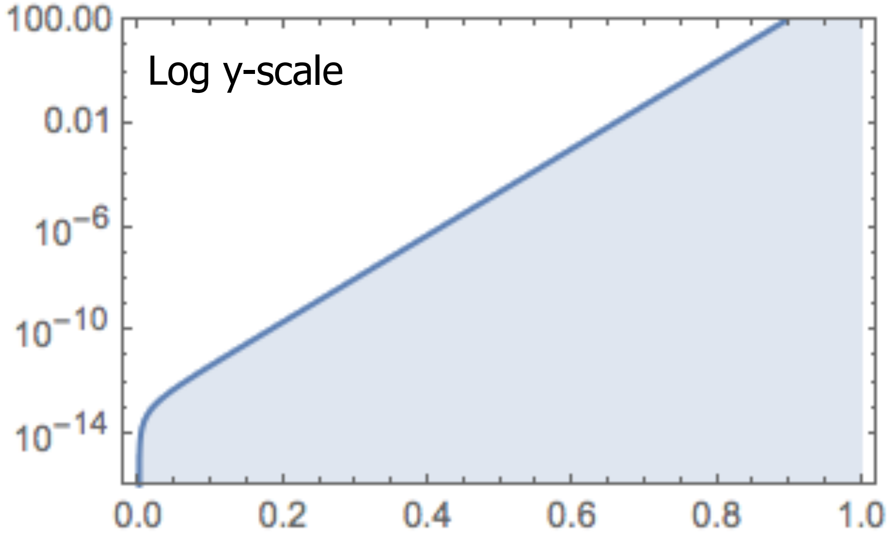
Applying an External Voltage

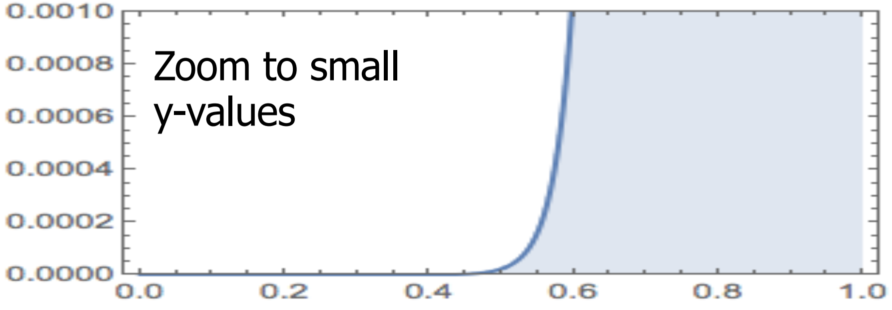
- An external voltage superimposes an additional field and thus *changes the drift contribution*
- The equilibrium *thickness is changed*
- When a positive voltage is applied to the p-side, the overall field is reduced, diffusion becomes stronger and ultimately an increasing current flows
  - To really understand this, solid state physics is required
- It turns out that $I_D = I_S(e^{U_D/U_{TH}} - 1)$
  - Diode current is exponential in a VERY wide range
  - U_D = Diode applied to device (relative to n-Side)
  - I_S = Saturation Current = Device property (mainly just size)
  - U_{TH} = Thermal Voltage = k T / q = 25.9mV @ RT

Diode Forward Current

- For I_S=0.1pA, U_{th} = 25.9 mV
- No magic ‘0.6V’ forward voltage, depends on 'scale'!



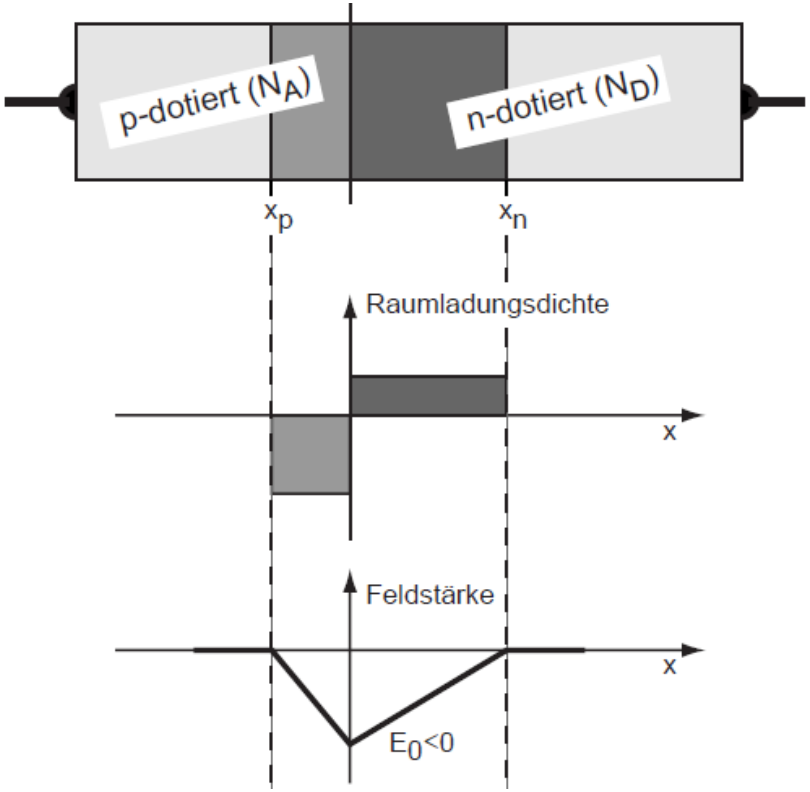




Thickness of Depleted Region (See also extra file)

- Charge on both sides must be equal:

$$\begin{aligned} Q_p &= -Q_n \\ A x_p q N_A &= -A x_n q N_D \\ x_p N_A &= -x_n N_D. \end{aligned}$$



- Field at junction:

$$E_{max} = \frac{q}{\epsilon} x_p N_A < 0.$$

- Potential = V_{bi}:

$$\Delta V = -\int_{x_p}^{x_n} E(x)dx = -\frac{1}{2}(x_n$$

$$= \frac{q}{2\epsilon}\frac{(N_A+N_D)N_A}{N_D}{x_p}^2$$

$$x_d = \sqrt{\frac{2\epsilon}{q}\frac{N_A+N_D}{N_A N_D}V_{bi}}\sqrt{1-\frac{V_{ext}}{V_{bi}}}$$

Dominated by low doped side! $\sqrt{\frac{2\epsilon}{q}\frac{V_{bi}}{N_D}}$

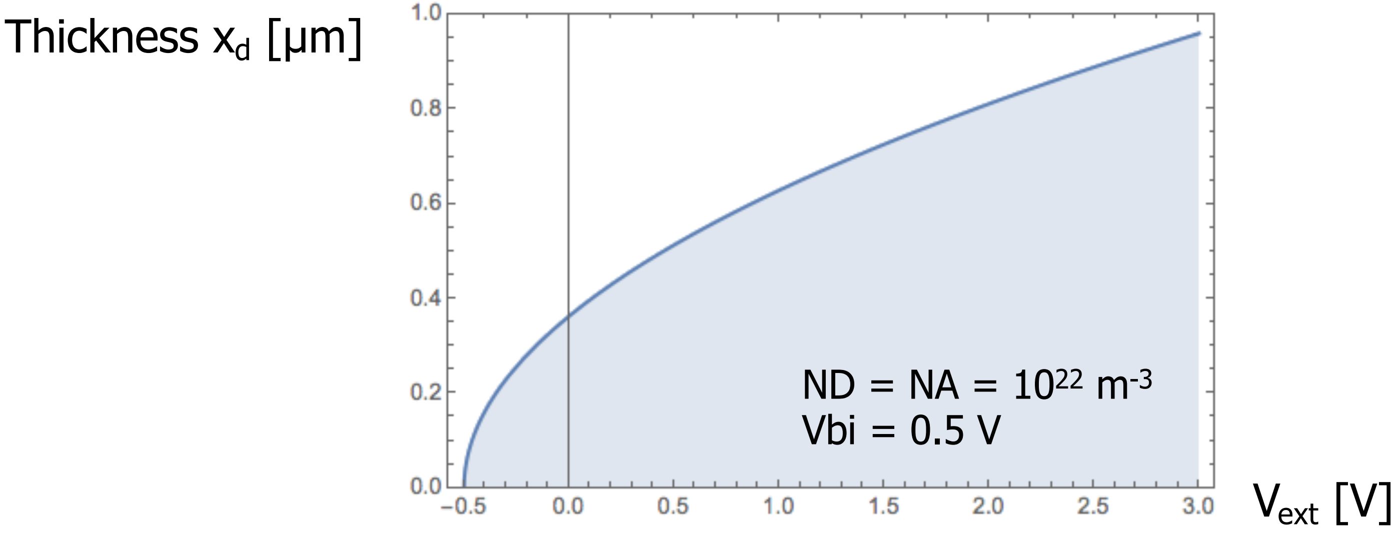
- Note: Depletion is thick for LOW doping

Dependence on External Voltage

- For the considered *abrupt* junction (p changes to n with no transition), we have

$$x_d \;=\; \sqrt{\frac{2\epsilon}{q}\,\frac{N_A+N_D}{N_A N_D}\,V_{bi}}\;\sqrt{1-\frac{V_{ext}}{V_{bi}}}$$

i.e. *the thickness of the depletion region increases as the square root of the external voltage* (for voltages ≫Vbi)



- Typical values on chips: $x_d \ll 1$ µm

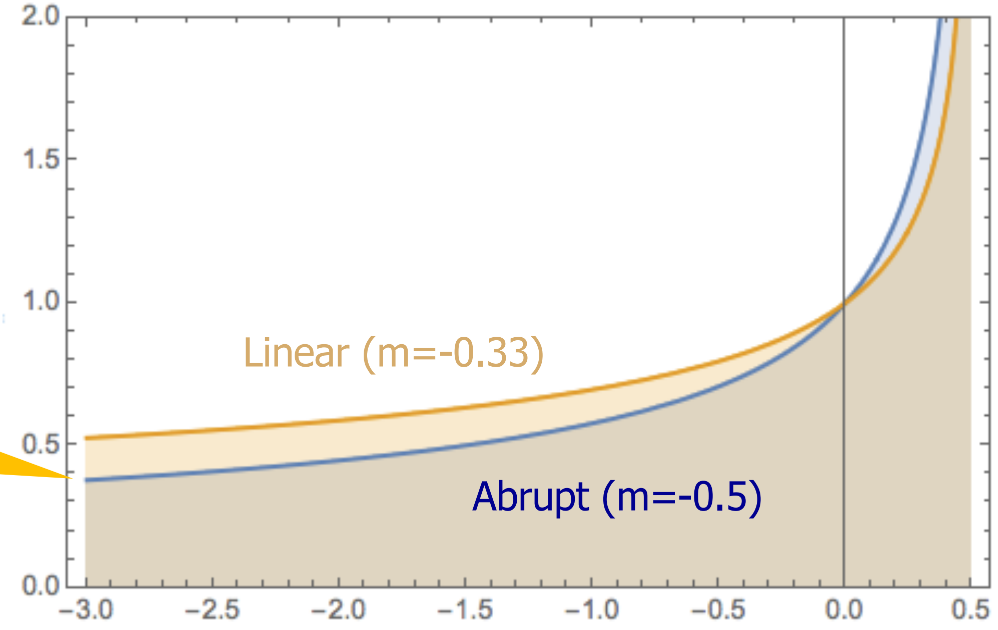
Capacitance

- The depletion region is a *parallel plate capacitor*

$$
\begin{aligned}
C_j &= \epsilon_0 \cdot \epsilon_{Si} \cdot \frac{A}{x_d} \\
\frac{C_j}{A} &= \underbrace{\sqrt{\frac{q\epsilon_0\epsilon_{Si}}{2}\frac{N_A N_D}{N_A + N_D}\frac{1}{V_{bi}}}}\left(1 - \frac{V_{ext}}{V_{bi}}\right)^{-1/2} \\
&= C_{j0} \cdot \left(1 - \frac{V_{ext}}{V_{bi}}\right)^{-1/2}
\end{aligned}
$$

Capacitance of parallel plate cap. of area A and thickness x_d, filled with (depleted) silicon

Exponent depends on doping profile:
-0.5 for abrupt junction
-0.33 for linear junction



Low capacitance for high (reverse) voltage

Diode Summary

- Diode is conducting when p-region is at positive voltage
- Forward current $I_D = I_S(\exp(V_D/U_T) - 1)$ increases x 10 every 60mV ($U_T = kT/q \sim 26\text{mV}$ @ 300K)

- E-Field is largest at the junction
- Potential increases quadratically (in constant doping)

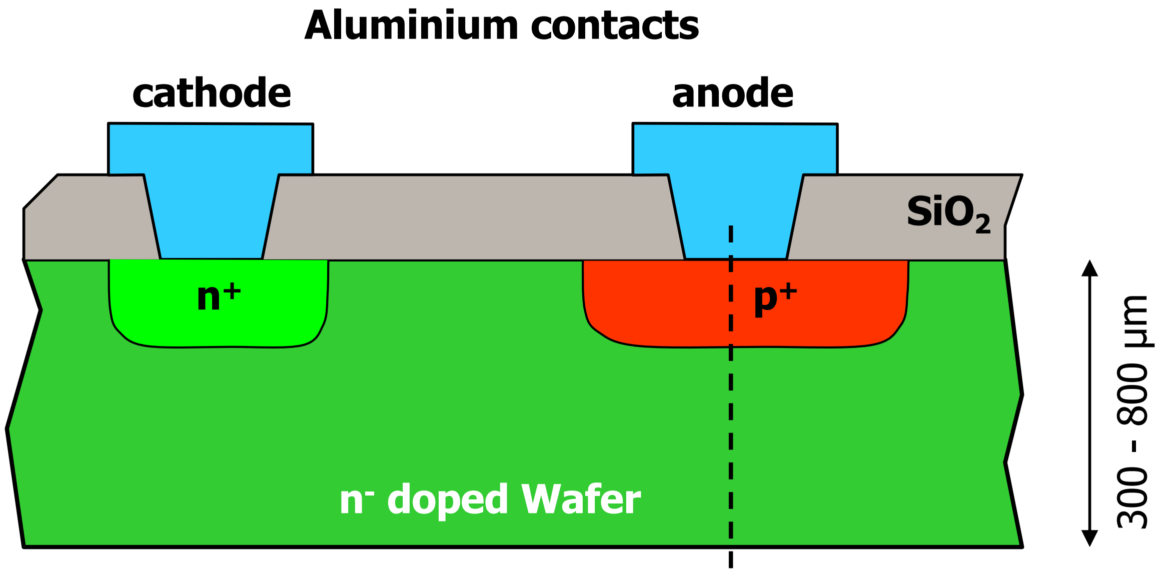
- Depletion region grows towards **low doped side**.
- Growth with √ of applied voltage $\sqrt{\frac{2\epsilon}{q}\frac{V_{bi}}{N_D}}$

- Capacitance decreases with 1/√ of applied (reverse) voltage

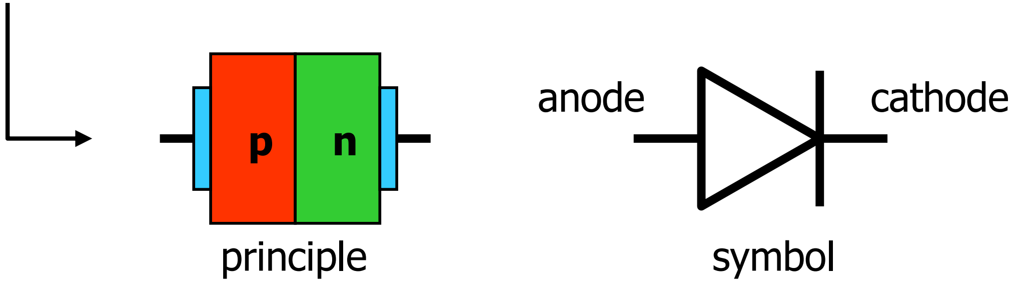
DIODE MODEL

A pn-Diode on a Chip Wafer

- For instance: n-doped Si 'Wafer' is p-doped at the surface
- EACH pn junction forms a diode



Cross section of an pn-junction on a wafer



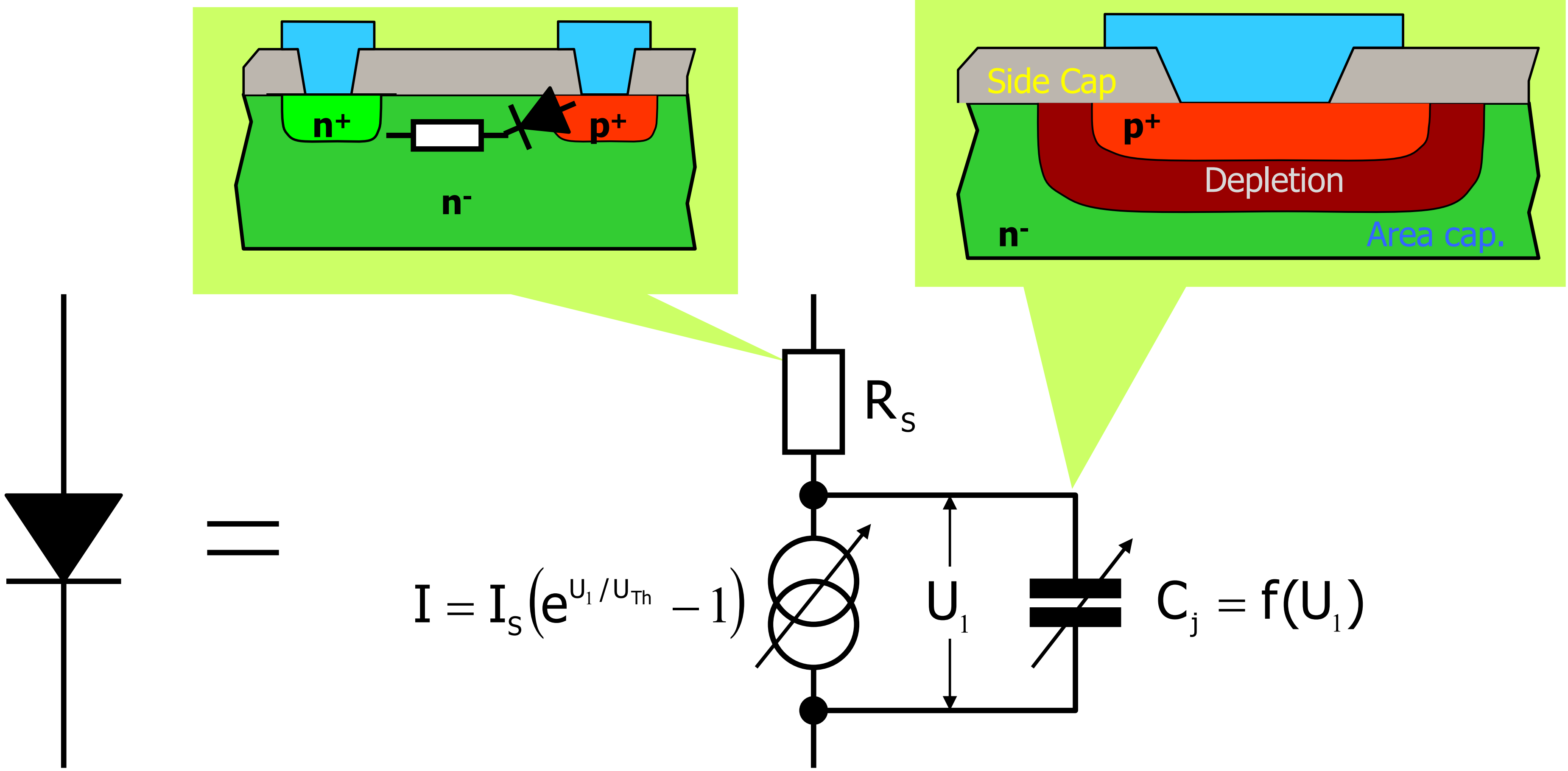
principle

symbol

Modell of the Diode

(mostly relevant for forward bias)

(mostly relevant for reverse bias)



$$I = I_S\left(e^{U_1/U_{Th}} - 1\right)$$

$$C_j = f(U_1)$$

Capacitance

- Is calculated by:

$$C(U) = A \cdot C_{j0} \cdot \left(1 - \frac{U}{V_{bi}}\right)^{-1/2} = A \cdot \mathbf{CJ0} \cdot \left(1 - \frac{U}{\mathbf{VJ}}\right)^{-\mathbf{M}}$$

- 3 Parameters:
  - **CJ0** : capacitance per unit area at U=0V
  - **VJ** : diffusion voltage = built-in-voltage
  - **m** : ‘grading coefficient’

- In more refined models, capacitance is the sum of
  - an AREA component (the bottom of the implant)
  - a SIDEWALL component (perimeter of the implant)

  Both contributions are added

$$C(U) = Area \cdot \mathbf{CJ0} \cdot \left(1 - \frac{U}{\mathbf{VJ}}\right)^{-\mathbf{M}} + Perimeter \cdot \mathbf{CJSW} \cdot \left(1 - \frac{U}{\mathbf{VJSW}}\right)^{-\mathbf{MSW}}$$

Important SPICE Parameters of the Diode

Parameter	Symbol	SPICE Name	Einheit	Default
Sättigungsstrom (Saturation current)	I_S	IS	A	1e-14
Serienwiderstand (Series resistance)	R_S	RS	Ohm	0
Sperrschichtkapazität bei VD=0V (Zero bias junction cap.)	C_{j0}	CJ0	F	0
Exponent in Kapazitätsformel (Grading Coefficient)	m	M	-	0.5
Diffusionsspannnung (Junction Potential)	Φ_0	VJ	V	1
Emissionskoeffizient (Emission Coefficient)	n	N	-	1
Transitzeit (Transit time)	τ_T	TT	s	0

- The values are for a unit size device. They are later multiplied by the diode AREA
- Transit time tells how long it takes for carriers to pass the depletion region.

Simple Small Signal Model

- Determine the slope at the working point:

$$I \approx I_S \cdot e^{U/U_{Th}}$$

$$\frac{\partial I}{\partial U} \approx \frac{1}{U_{Th}} \cdot I_S \cdot e^{U/U_{Th}} = \frac{I}{U_{Th}}$$

$$\Rightarrow R_{eq}(U_0) = \frac{U_{Th}}{I_0}$$