The Diode

Formula Collection

Field	E		points from positive to negative charges (electrons flow against field lines)
	E	$= -\,\mathrm{grad}\,\Psi = -\,dV(x)\,/\,dx$	
Potential	Ψ	$= -\int E(x')\,dx'$	
Maxwell-eq.:	$\int E(x)\,dA$	$= Q\,/\,\varepsilon\varepsilon_0$	'Integral of fields = included charge'
Gauss law:	$\mathrm{div}\,E$	$= \rho\,/\,\varepsilon\varepsilon_0$	= differential form
Poisson eq.:	$\partial^2\Psi/\,\partial x^2$	$= \rho\,/\,\varepsilon\varepsilon_0$	= Gauss law
Laplace eq.:	$\partial^2\Psi/\,\partial x^2$	$= 0$	= Poisson equation in empty space
Current density:	$j(x)$	$= -\,\sigma\,E(x)$	($[j] = A/m^2$, $[E] = V/m$, $[\sigma] = A/(Vm) = S/m$)
conductivity:	σ	$= q\,n\,\mu$	(n: carrier density, q: charge, μ: mobility) ($[q] = C$, $[n] = m^{-3}$, $[\mu] = m^2/Vs$)
resistance	$\rho[\Omega m]$	$= E/j = 1/\sigma$	($R = \rho\, l/A$, l=length, A=area)

A few constants

q	1.602×10^{-19} C	elementary charge
k	1.381×10^{-23} J/K	Boltzmann constant
4kT	1.657×10^{-20} J	Noise Power density @ 300K
U_T	$= kT/q = 25.9$ mV	Thermal voltage @ 300K
ε_0	8.854×10^{-12} F/m	vacuum susceptibility (Hint: $C = \varepsilon_0 A/d$, 1m x 1m x 1m: ~10pF)

A few constants for silicon

E_g	1.12	eV	band gap at 300K
N_{atom}	5×10^{22}	cm^{-3}	atom density
N_i	1.01×10^{10}	cm^{-3}	intrinsic carrier density at 300K* (‘old’ value: 1.45)
μ_e	1400	cm^2/Vs	electron mobility (@ low fields)
μ_h	480	cm^2/Vs	hole mobility ($v = \mu E$)
E_{cit}	1	V/µm	critical field where mobility starts to drop
ε_{Si}	11.9		dielectric constant silicon
ε_{SiO2}	3.90		dielectric constant silicon - dioxide
E_{max}	$\sim 3 \times 10^7$	V/m	break through field strength
E_{eh}	3.6	eV	Av. Energy required to generate an e-h pair
ρ	7.87	gcm^{-3}	density
λ	150	W / (m · K)	thermal conductivity
α	2.6–3.3	K^{-1}	thermal expansion coefficient (Aluminum: 22-25)

- *Sproul AB, Green MA. Improved value for the silicon intrinsic carrier concentration from 275 to 375 K. Journal of Applied Physics [Internet]. 1991;70:846-854. Available from: http://link.aip.org/link/?JAP/70/846/1

Energy Bands / Doping ⇒ Applets on SuS web site

- Silicon-crystal
- Creation of bands, Valence & Conduction band
- Density of States
- Fermi distribution
- Intrinsic Carrier density n_i
- Hole conduction

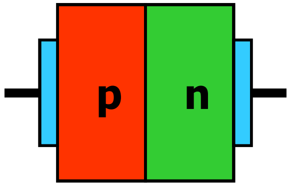
- N - doping (e.g. phosphorous, arsenic, $10^{14\text{-}20}\text{cm}^{-3}$)
- P - doping (e.g. boron)
- Location of donator / acceptor Energy levels

- Mass action law: $n \times p = n_i^2$

- Fermi – level in doped semiconductor
- n - Doping ⇒ E_F is moved towards conduction band
- p - Doping ⇒ E_F is moved towards valence band

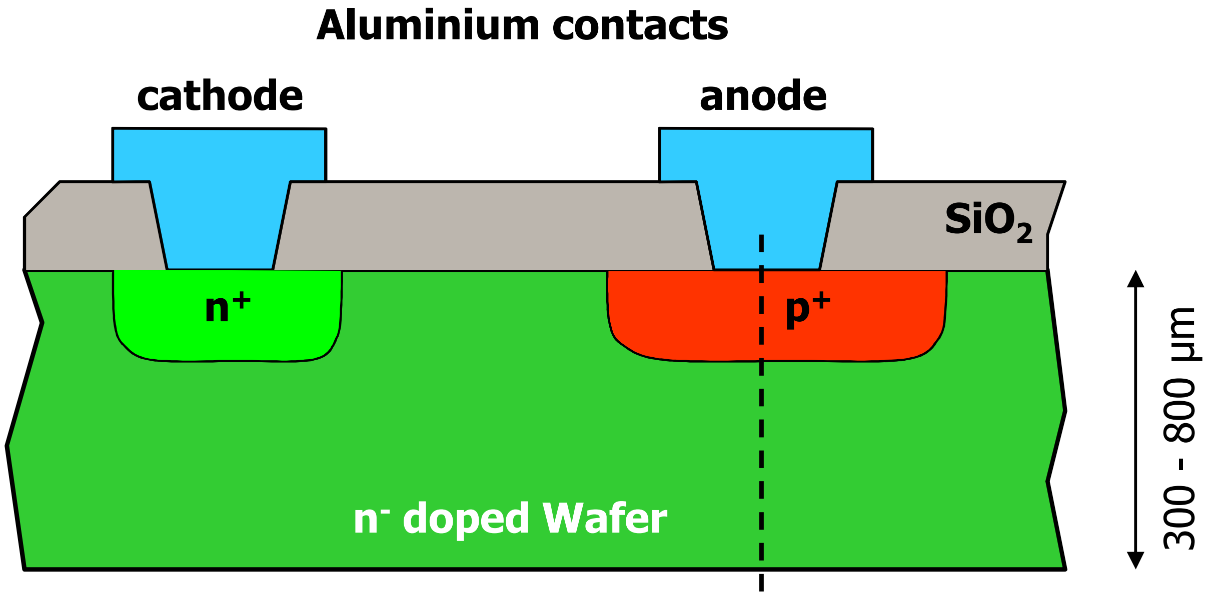
- Strength of doping is marked with exponent + or –, for instance n^-, p^{++}

The Diode (p-n-junction)

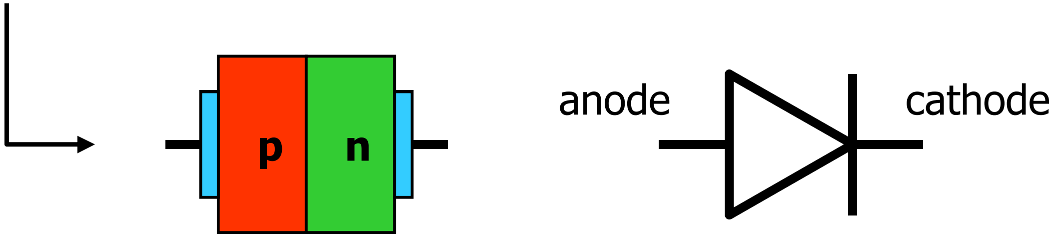


pn-Diode on wafer

- For instance: n-doped Si 'Wafer' is p-doped at the surface
- EACH pn junction forms a diode



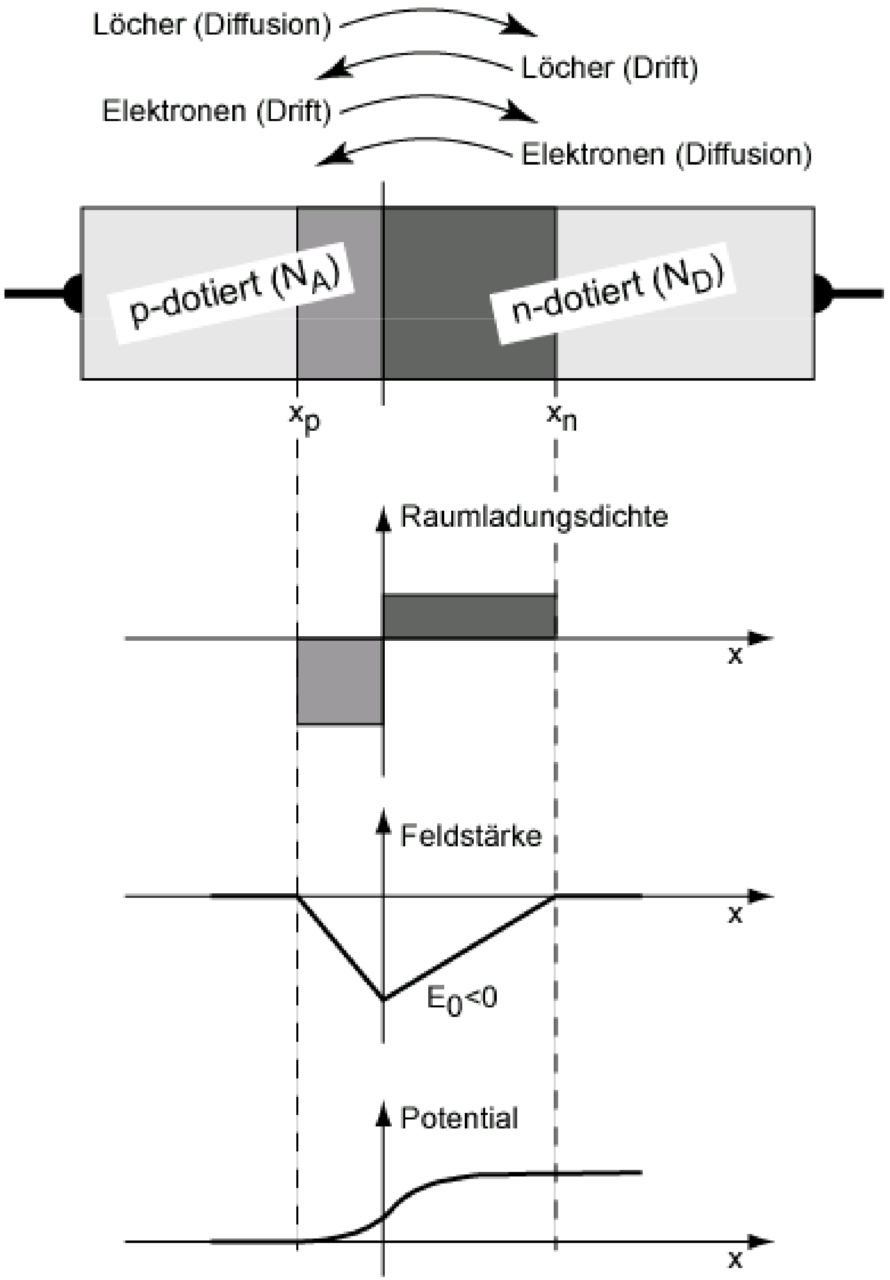
Cross section of an pn-junction on a wafer



principle

symbol

Origin of Depletion Layers



- We consider an idealized, 'abrupt' transition between n- and p- region (this is smooth in reality)

- Due to the concentration gradient, electrons diffuse from the n → p region (holes from p → n).
- The carriers compensate and we get depleted regions without mobile carriers

- The fixed, ionized atoms are positively charged in the n-region (negatively in the p-region)
- This leads to an electric field
- The field is associated with a electrostatic potential. This ‘built in' potential depends only on doping.
- The field leads to a drift of electrons/holes backwards.
- The thickness of the depletion region is determined by the equilibrium between drift- and diffusion currents

- In reality, the depletion zone drops more slowly to zero, but the transition regions is small.

Build-In Voltage

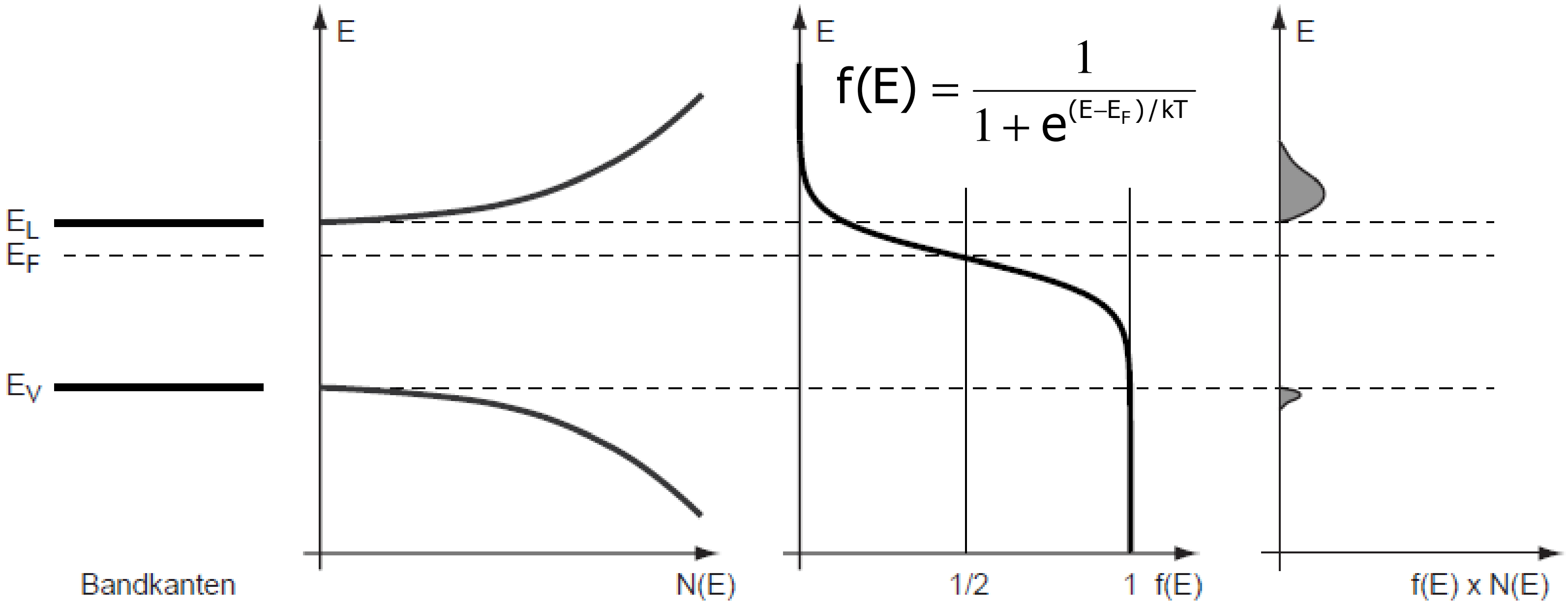
- Derivation steps (p(x)= hole density):

$$\begin{aligned}
j_{Feld}(x) &= -j_{Diff}(x) \\
q\,\mu\,p(x)E(x) &= q\,D\frac{dp(x)}{dx} \\
-\frac{q}{kT}dV(x) &= \frac{dp(x)}{p(x)} \\
-\frac{q}{kT}\int_{V_p}^{V_n} dV(x) &= \int_{p_p}^{p_n}\frac{dp(x)}{p(x)} \\
-\frac{q}{kT}(V_n - V_p) &= \ln\left(\frac{p_n}{p_p}\right) \\
V_{bi} := V_n - V_p &= \frac{kT}{q}\ln\left(\frac{p_p}{p_n}\right)
\end{aligned}$$

$$\begin{aligned}
V_{bi} &= \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) \\
&\approx \left[\log\left(\frac{N_A}{n_i}\right) + \log\left(\frac{N_D}{n_i}\right)\right] \times 60\,\mathrm{mV}
\end{aligned}$$

Derivation using Energy Bands

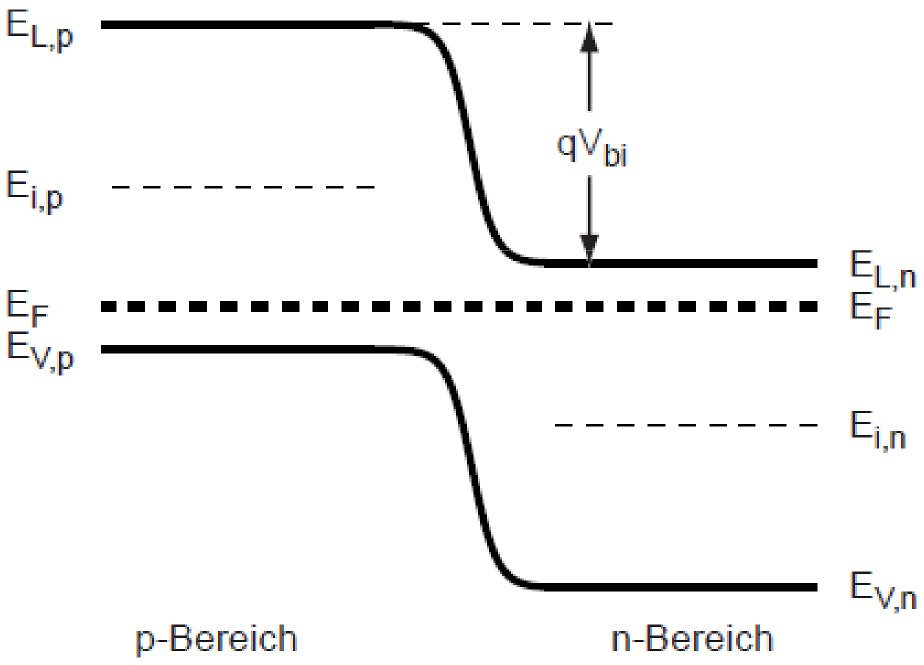
- Fermi Distribution + Density of States



$$\begin{aligned}
n &= \int_{E_L}^{\infty} N(E) f(E) dE \\
&= \underbrace{2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}} e^{(E_F-E_L)/kT} \\
&= \qquad N_C \quad \cdot \quad e^{(E_F-E_L)/kT}
\end{aligned}$$

$$\begin{aligned}
E_{F,n} &= E_L - kT \ln \frac{N_{C,n}}{N_D} \\
E_{F,p} &= E_V + kT \ln \frac{N_{C,p}}{N_A}
\end{aligned}$$

Derivation using Energy Bands



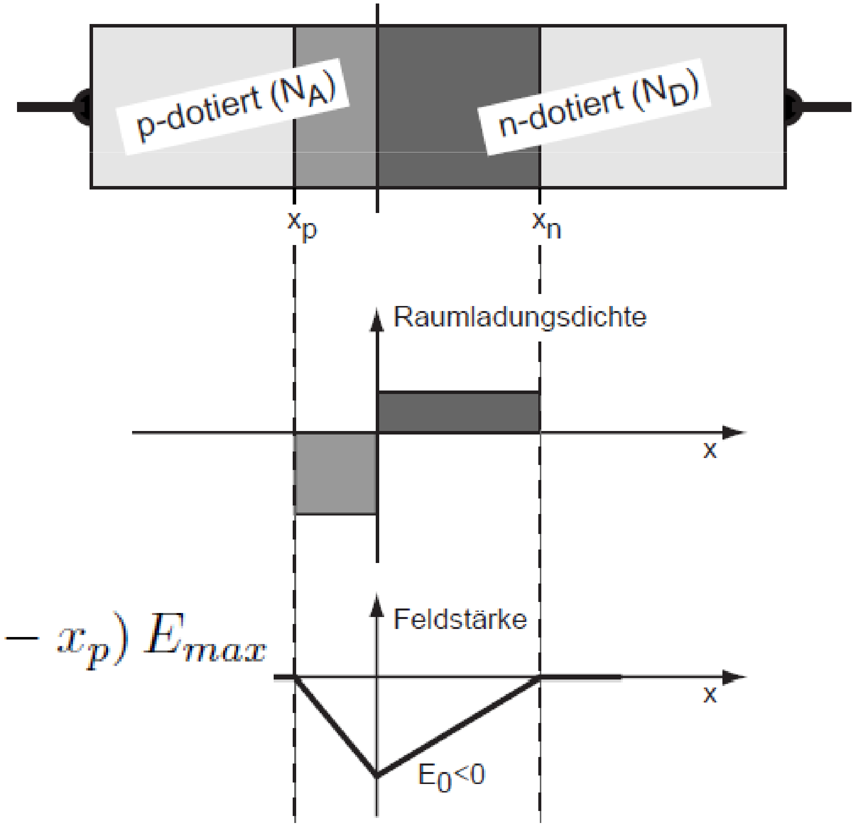
$$
\begin{aligned}
qV_{bi} &= E_{L,p} - E_{L,n} \\
&= \underbrace{E_{L,p} - E_{V,p}} + \underbrace{E_{V,p} - E_F} + \underbrace{E_F - E_{L,n}} \\
&= kT \ln \frac{N_{C,n} N_{C,p}}{n_i^2} - kT \ln \frac{N_{C,p}}{N_A} - kT \ln \frac{N_{C,n}}{N_D}
\end{aligned}
$$

$$
V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i n_i}
$$

Thickness of Depleted Region

- Charge on both sides must be equal:

$$\begin{aligned} Q_p &= -Q_n \\ A x_p q N_A &= -A x_n q N_D \\ x_p N_A &= -x_n N_D. \end{aligned}$$



- Field at junction:

$$E_{max} = \frac{q}{\epsilon} x_p N_A < 0.$$

- Potential = V_{bi}:

$$\begin{aligned} \Delta V &= -\int_{x_p}^{x_n} E(x)dx = -\frac{1}{2}\left(x_n - x_p\right) E_{max} \\ &= \frac{q}{2\epsilon} \frac{(N_A + N_D)N_A}{N_D} {x_p}^2 \end{aligned}$$

$$x_d = \sqrt{\frac{2\epsilon}{q} \frac{N_A + N_D}{N_A N_D} V_{bi}} \sqrt{1 - \frac{V_{ext}}{V_{bi}}}$$

Dominated by low doped side! $\sqrt{\frac{2\epsilon}{q} \frac{V_{bi}}{N_D}}$

Example

- Detector silicon is **weakly** doped, i.e. **high resistivity**
- For instance: 5 kΩ · cm, n-doped (very high res!)

- $N_D = (q\,\mu\,\rho)^{-1}$
  $= (1.6 \times 10^{-19}\,\text{As}\ 1400\,\text{cm}^2/\text{Vs}\ \mathbf{5k\Omega\ cm})^{-1}$
  $\sim \mathbf{10^{12}\,cm^{-3} = 1\ per\ \mu m^3}$
- Assume for instance $N_A = 10^{16}\,\text{cm}^{-3}$ ($n_i = 1.45 \times 10^{10}\,\text{cm}^{-3}$)
- $V_{bi} \sim 60\text{mV} \times [\log(N_A/n_i) + \log(N_D/n_i)] \sim 60\text{mV} \times (6+2) \sim \mathbf{480mV}$

- Depletion thickness:

$$D = \sqrt{\frac{2\epsilon}{q}\frac{V_{bi}}{N_D}} \sim 25\,\mu\text{m}$$

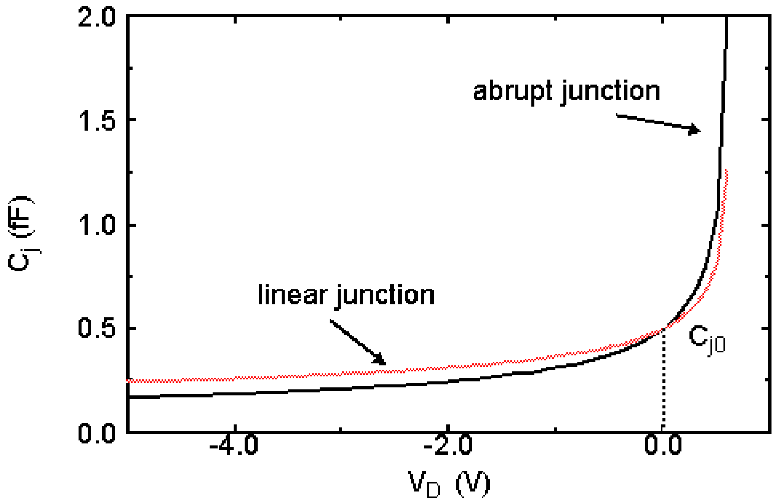
- $D \rightarrow 2\ D$ for $V_{ext} = 3 \times V_{bi} = 1.5\ \text{V}$
- $D \rightarrow 10D$ for $V_{ext} = 99 \times V_{bi} = 47\ \text{V}$

Capacitance

- Large detectors are parallel plate capacitors

$$
\begin{aligned}
C_j &= \epsilon_0 \cdot \epsilon_{Si} \cdot \frac{A}{x_d} \\
\frac{C_j}{A} &= \underbrace{\sqrt{\frac{q\epsilon_0\epsilon_{Si}}{2} \frac{N_A N_D}{N_A + N_D} \frac{1}{V_{bi}}}} \left(1 - \frac{V_{ext}}{V_{bi}}\right)^{-1/2} \\
&= C_{j0} \cdot \left(1 - \frac{V_{ext}}{V_{bi}}\right)^{-1/2}
\end{aligned}
$$

Depends on doping profile

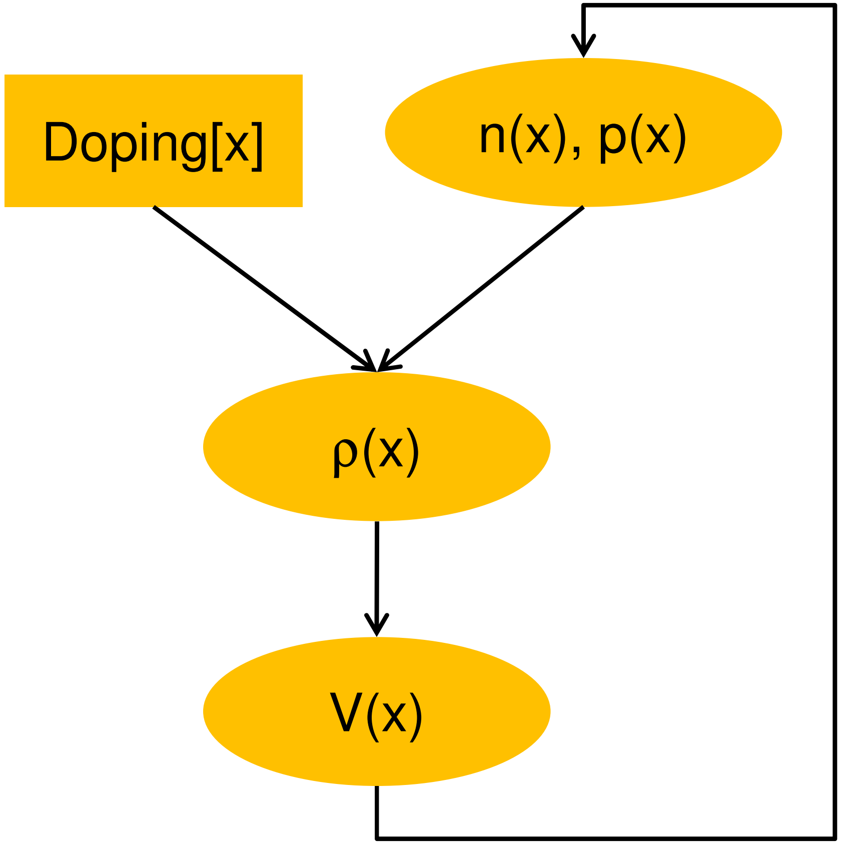


$$C_j = \frac{C_{j0}}{(1 - V_D / \phi_0)^m}$$

m = 0.5: abrupt junction
m = 0.33: linear junction

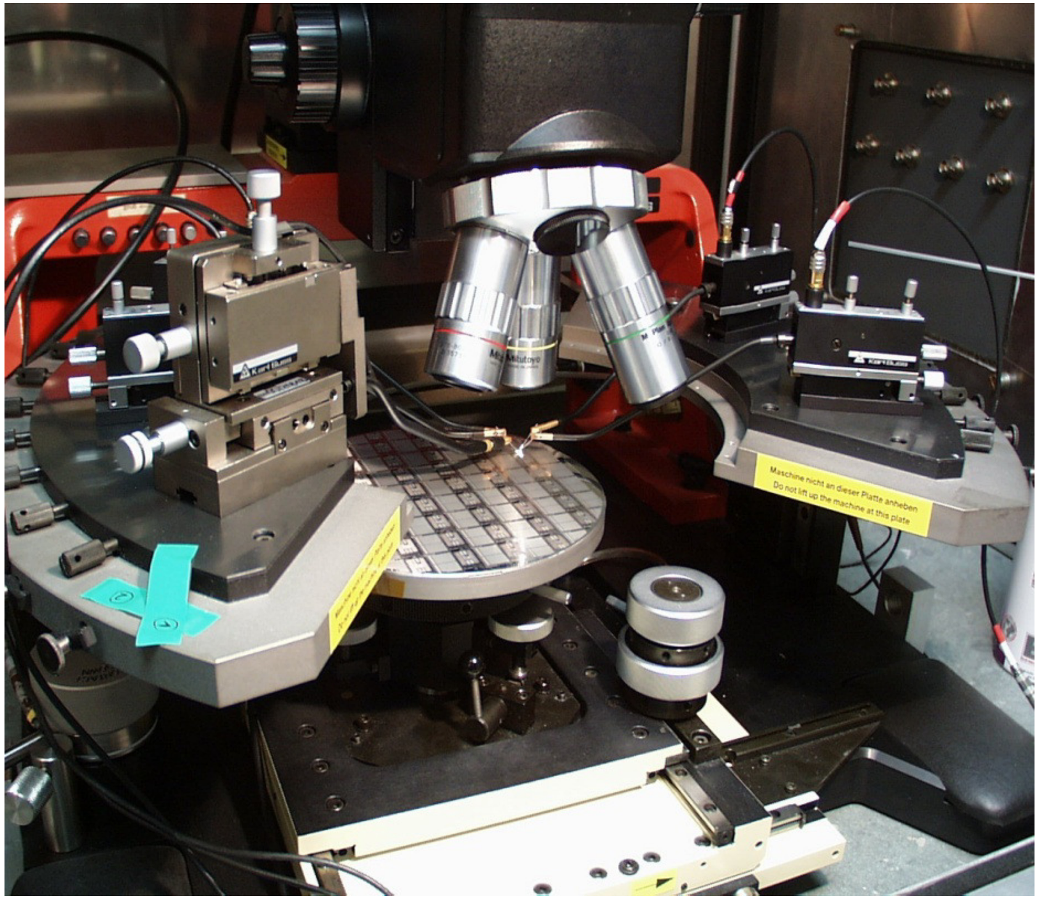
General Numerical Solution

- See .pdf document + Demonstration Program



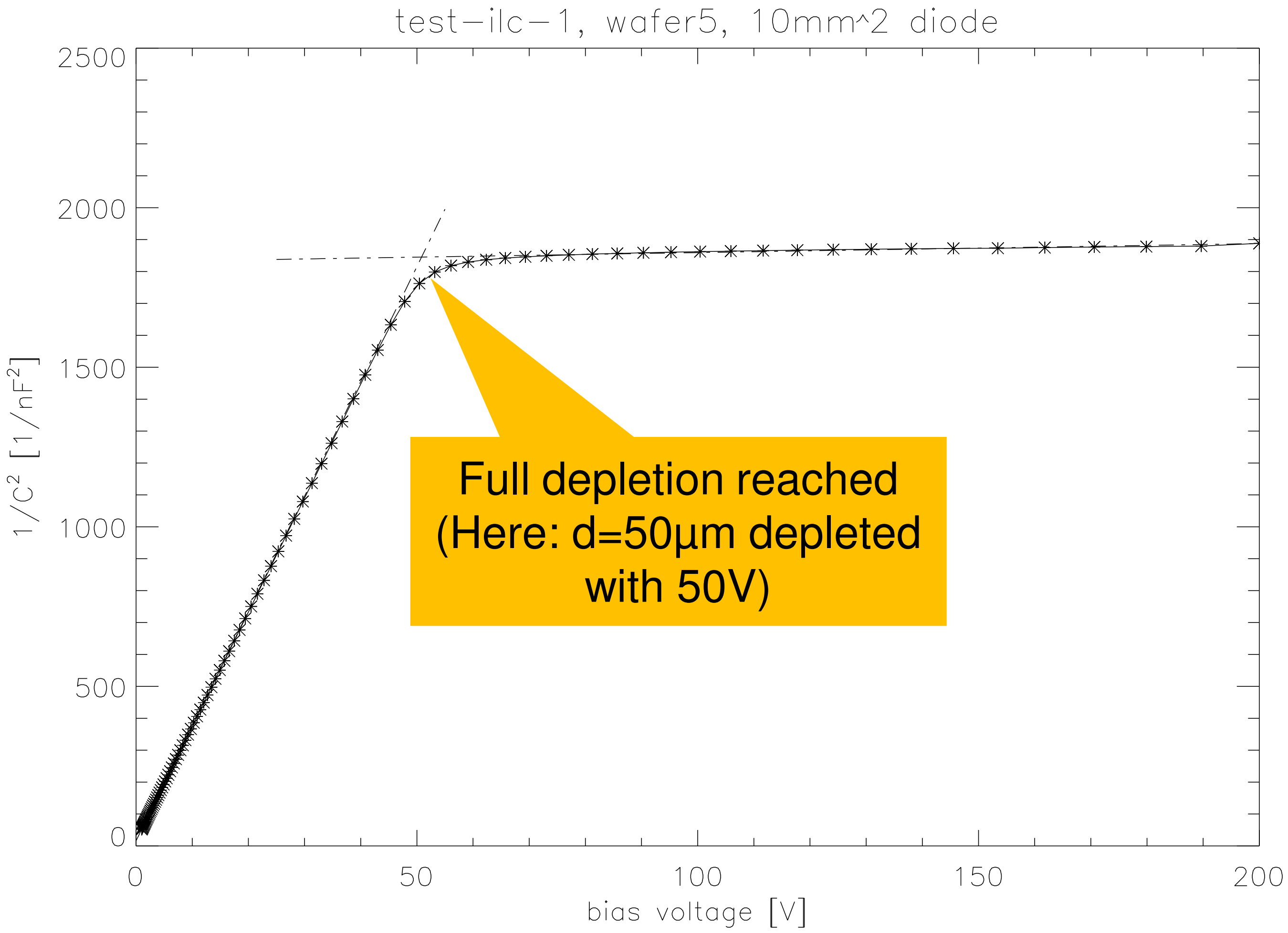
C(V) or I(V) Measurement Setup

- Contacting diode on wafer with **probe station** (in dark box)
- Bias + Current measurement with Source-Monitoring-Unit (SMU) and CV Meter



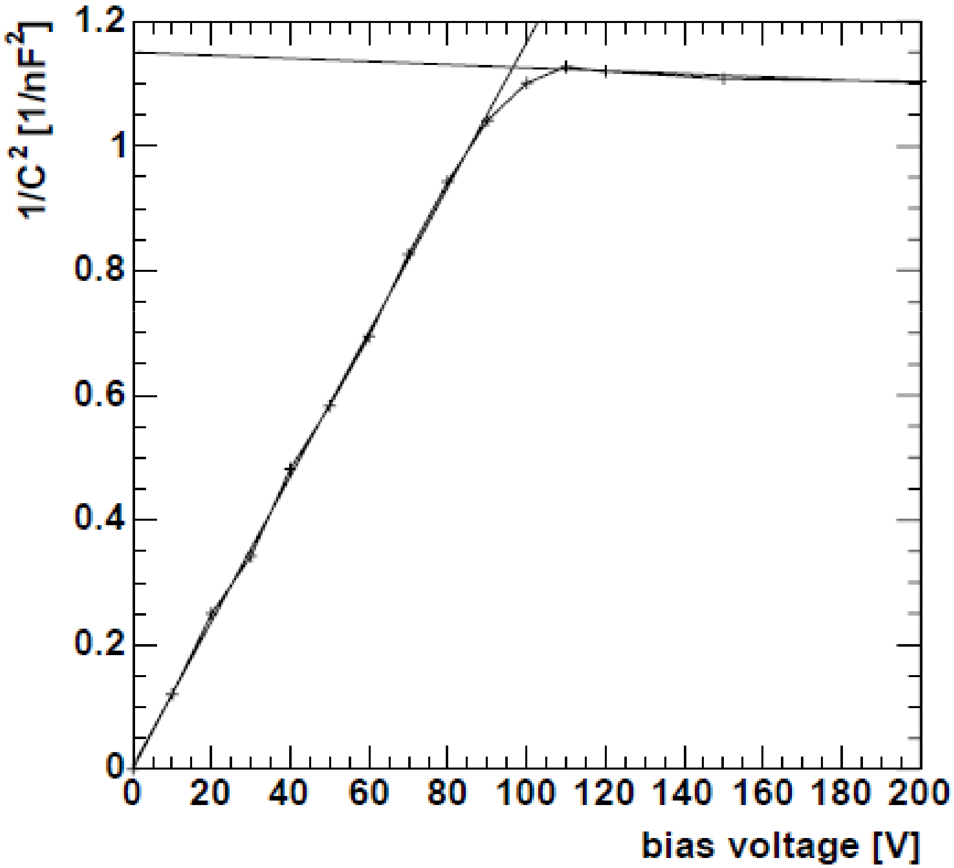
Measurement: CV-curve

- If C ~ $V^{-1/2}$, then $1/C^2$ ~V
- Textbook measurement (HLL Munich):



Full depletion reached
(Here: d=50µm depleted
with 50V)

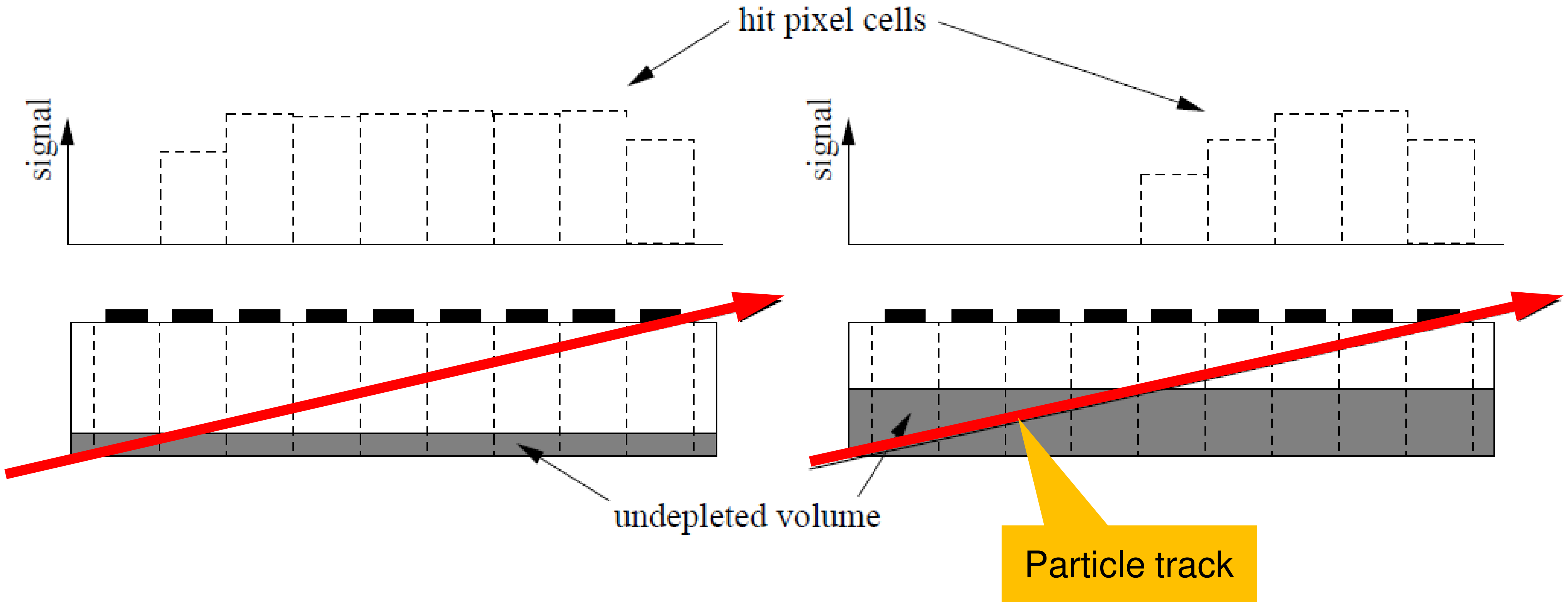
Similar measurement for a 300µm thick detector



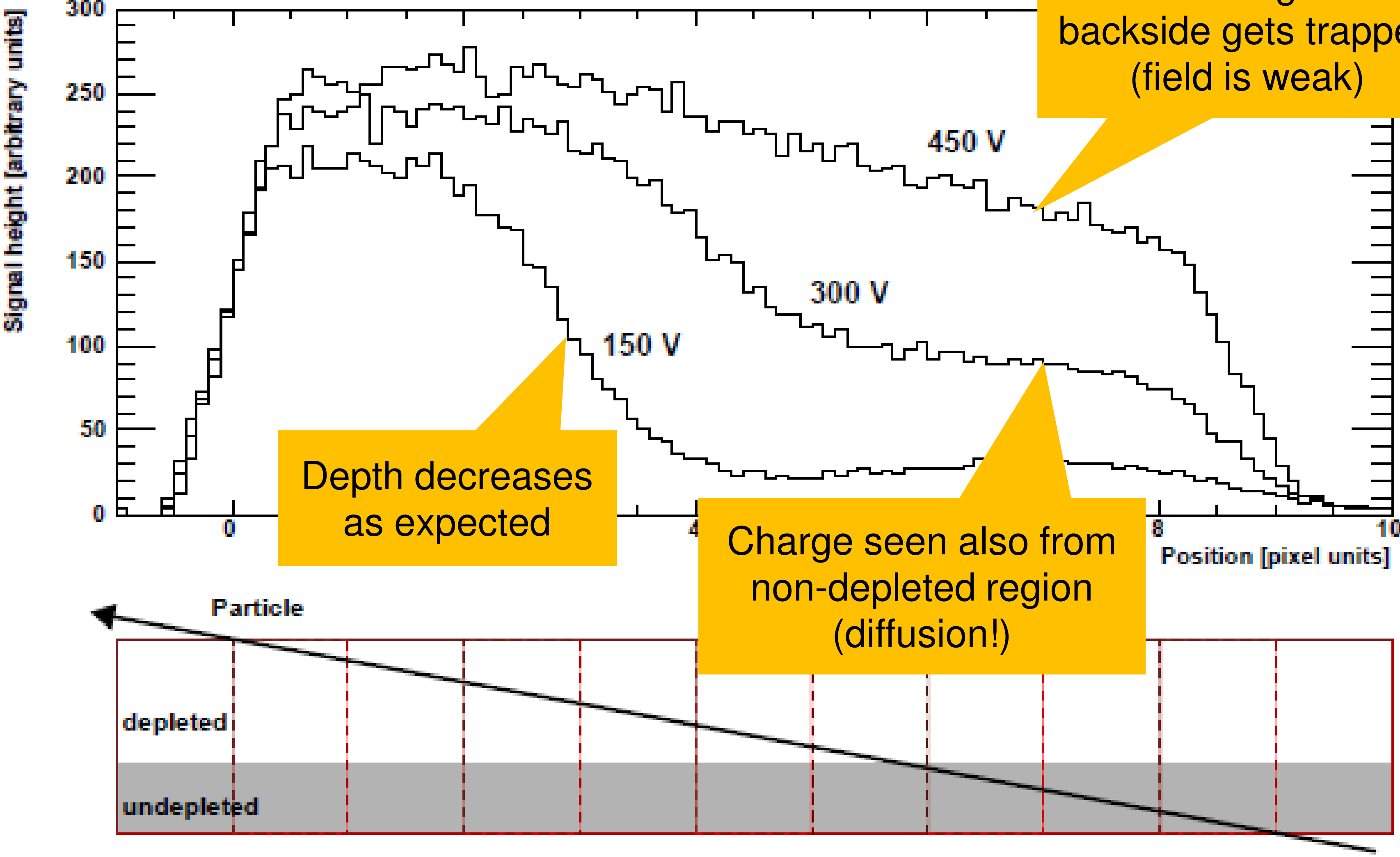
$$C(V) = \frac{\varepsilon_0 \varepsilon_{\mathrm{Si}}}{W(V)} \approx \begin{cases} \sqrt{\dfrac{\varepsilon_0 \varepsilon_{\mathrm{Si}} e N_{\mathrm{D}}}{2V}} & \text{for } V < V_{\mathrm{depl}} \\ \dfrac{\varepsilon_0 \varepsilon_{\mathrm{Si}}}{d} & \text{for } V > V_{\mathrm{depl}} \end{cases}$$

Direct Measurement of partial depletion

- Clever measurement with particle beam (R. Horisberger):
- At **very shallow beam incidence**, the number of hit strips is smaller than the projection if depletion is partial



Measurement



Some charge from backside gets trapped (field is weak)

Depth decreases as expected

Charge seen also from non-depleted region (diffusion!)

‘Leakage Current’

- Even in a ‚perfect‘ detector, eh pairs are generated by thermal excitation

- This **leakage current** is
  - Proportional to the depleted **volume**
  - Increases with temperature, for instance: $I_L \propto T^2 \, \mathrm{Exp}(-E_g/2kT)$
  - There are sometimes ‘surface effects’
- Cooling from 20°C to 6°C reduces leakage to ~1/10

- Electron/hole emission & capture (→ leakage) are eased by
  - Impurity atoms
  - Crystal defects
  - Radiation damage (displaced atoms)
- Detector production must be very clean and careful

Breakdown

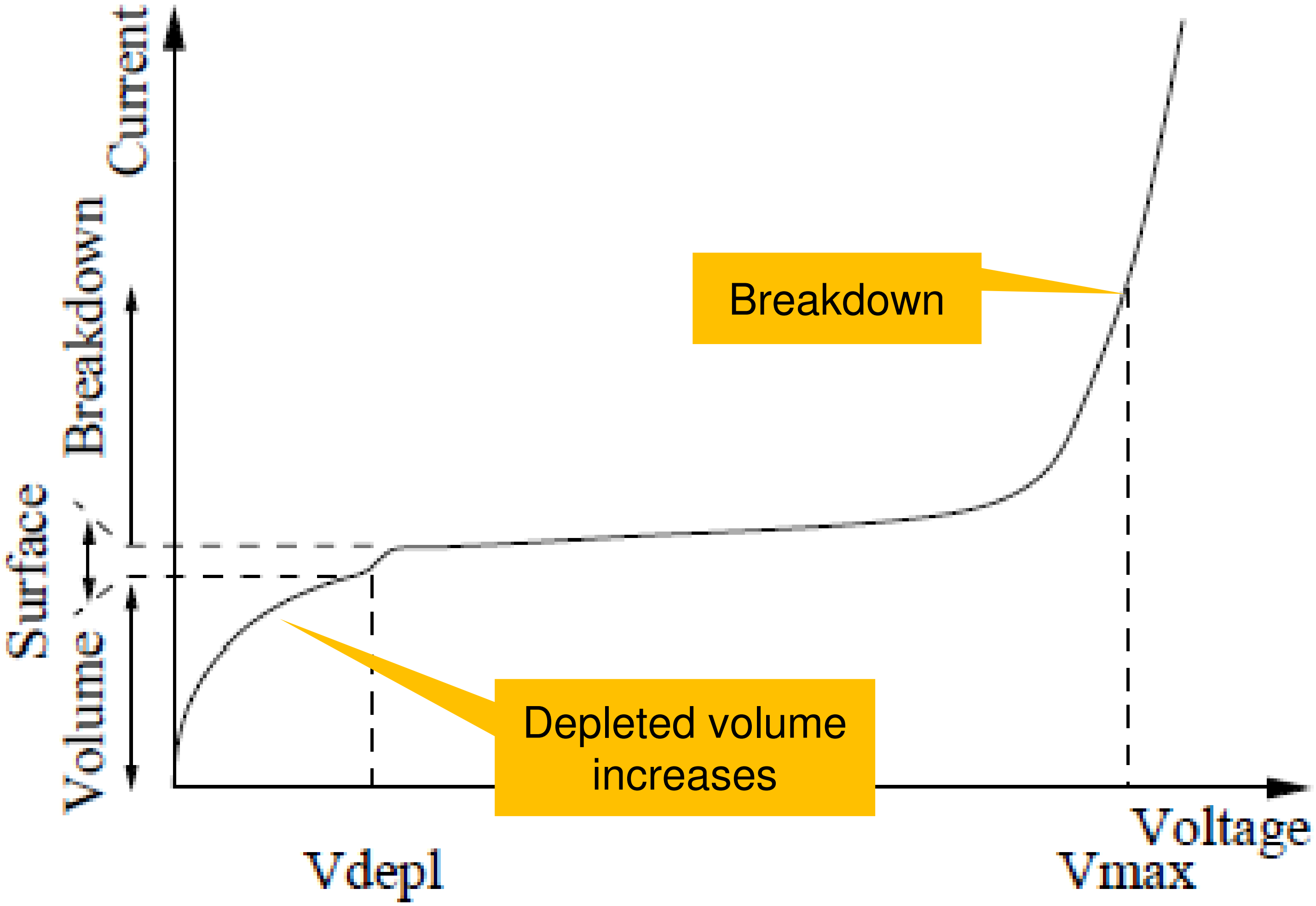
- At very high reverse voltages, diode can ‘break down’:

- E-Field is above **critical value**
- Fields are so high that single (leakage) electrons generate secondary electrons → avalanches

- Breakdown can occur
  - at local (point) defects
  - at points with high field strengths (strong doping, edges)

- The local current heats the detector
  → current increases
  → spot gets hotter
  → …
  This situation is called ‘thermal runaway’

Ideal I-V characteristic

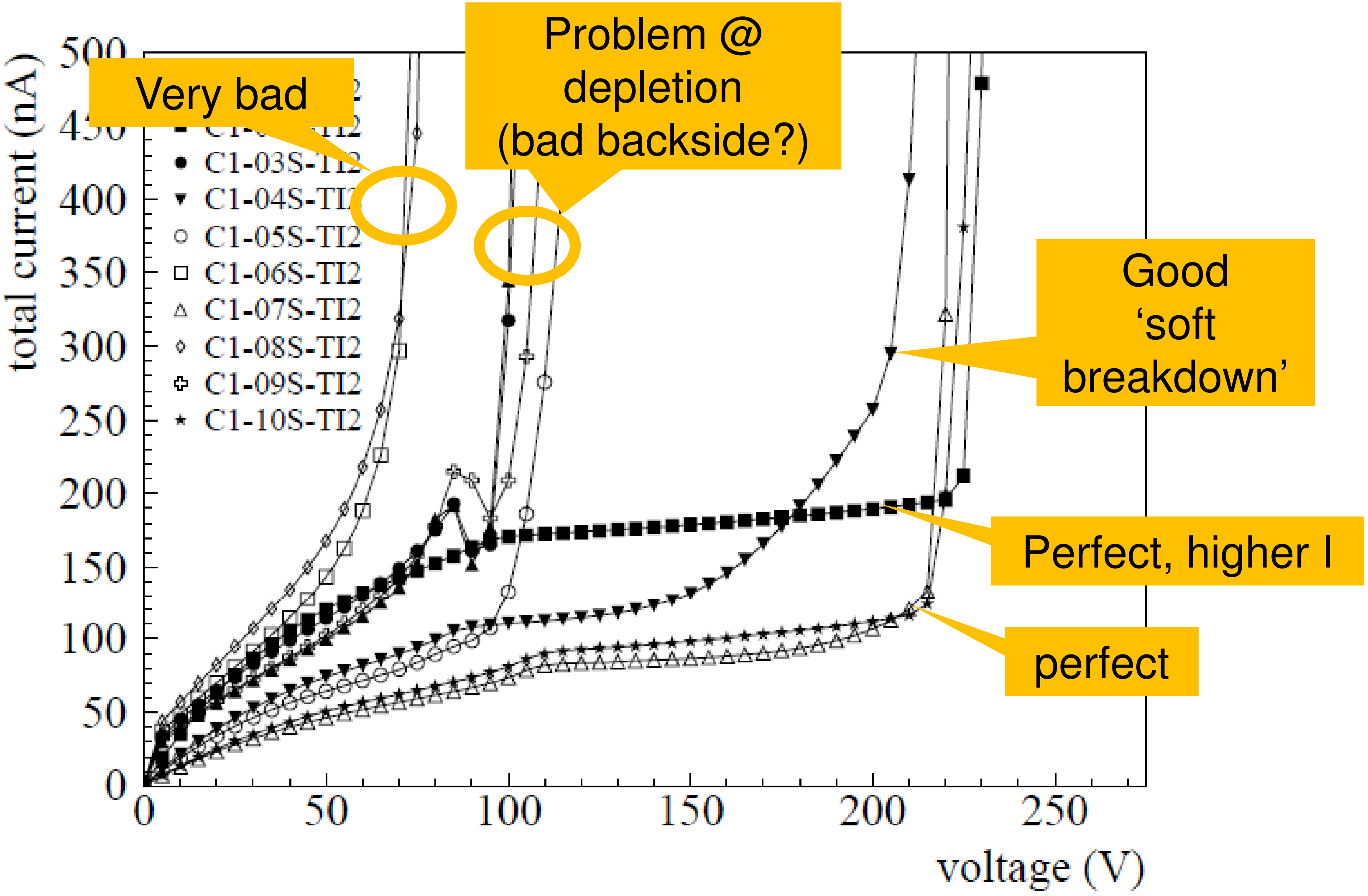


Breakdown

Depleted volume increases

Real Large Area Detectors

- (ATLAS) Pixel detectors of 9 cm² area.



Very bad

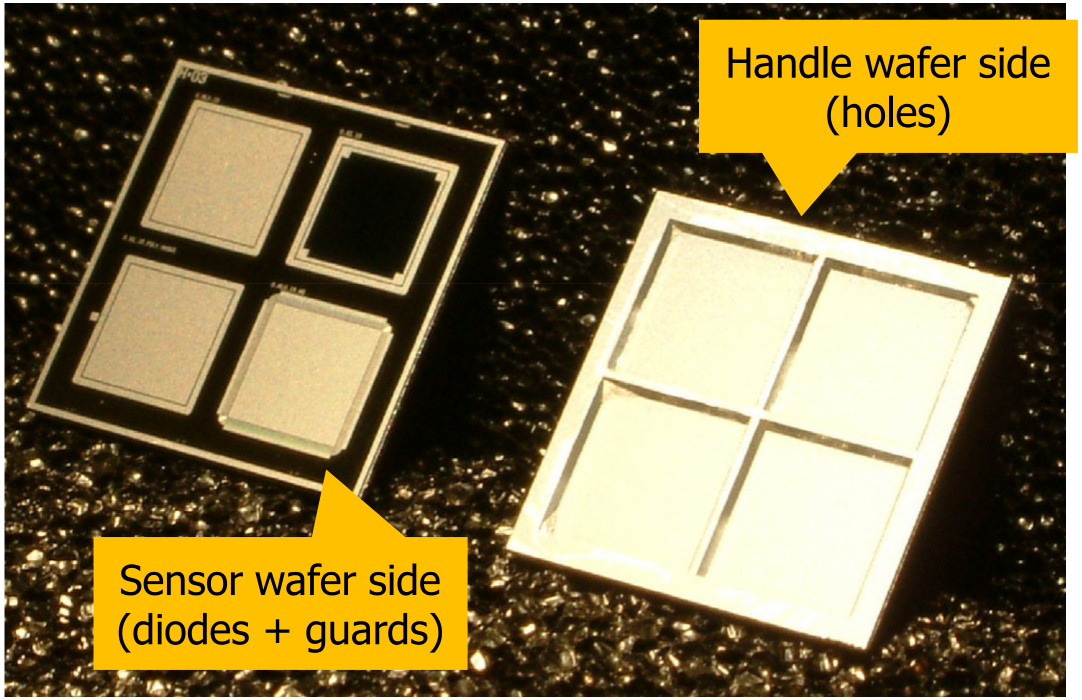
Problem @ depletion (bad backside?)

Good ‘soft breakdown’

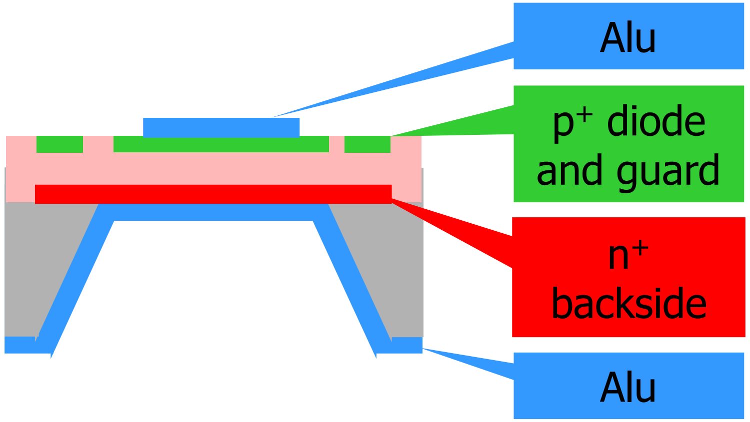
Perfect, higher I

perfect

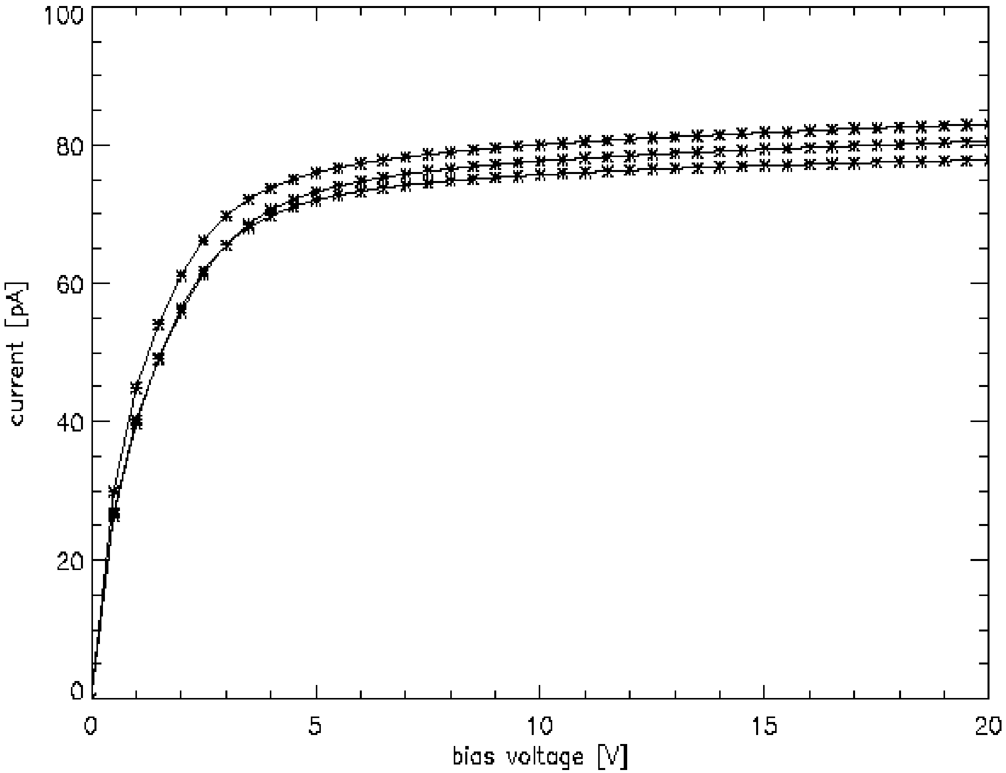
IV-Measurements


Handle wafer side (holes)

Sensor wafer side (diodes + guards)



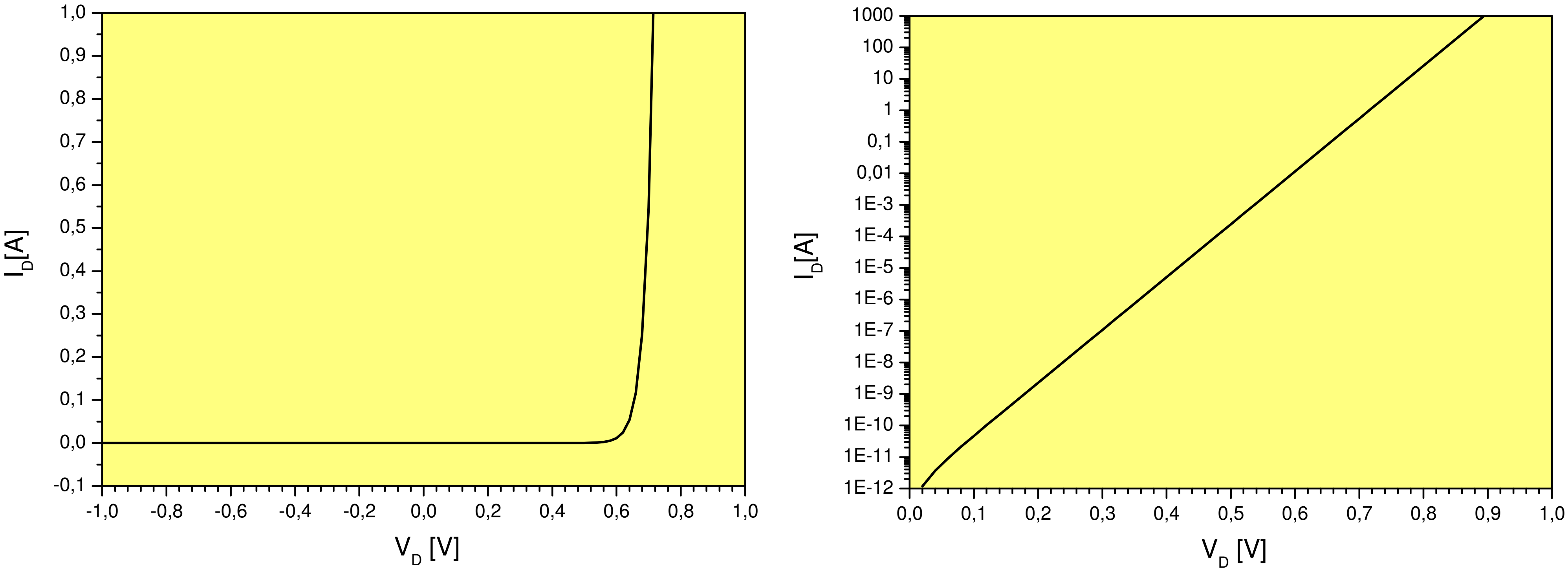
- 10mm² diodes of t=50µm
- Measured leakage currents are very low: **150 nA/cm³** (~very good strip detector)
- No breakdown is observed even at strong over-depletion



Diode Forward Current

- Diode Forward Current is exponential
- No magic ‘0.6V’ forward voltage!

$$I_D = I_S\left(e^{U_D/U_{TH}} - 1\right)$$



Diode Summary

- Diode is conducting, when p-region is at positive voltage
- Forward current $I_D = I_S(\exp(V_D/U_T) - 1)$.
  ($U_T = kT/q \sim 26mV$ @ 300K). I increases x 10 every 60mV

- E-Field is largest at the junction
- Potential increases quadratically (in constant doping)

- Depletion region grows towards **low doped side**.
- Growth with √ of applied voltage
- Capacitance decreases

$$\sqrt{\frac{2\epsilon}{q}\frac{V_{bi}}{N_D}}$$

- When n-doped regions are depleted, they charge positive!
  - E-field can point to one point from all sides!
  - Electrons are attracted by depleted n-regions

Diode as a Detector

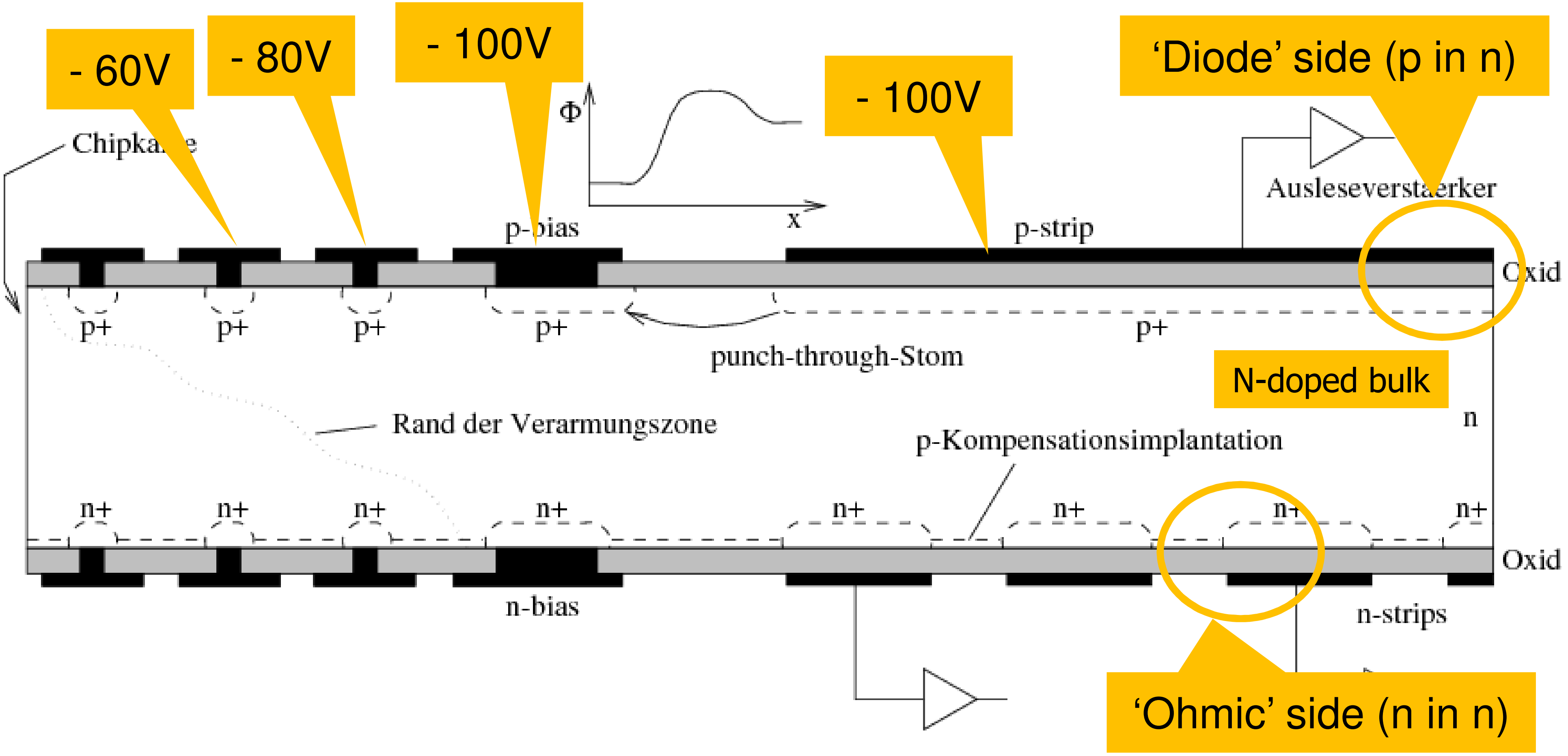
- Typical thickness ~ 300µm (standard 6'' wafer material)

- Can use
  - p implants in n bulk 'p on n' ← more common
  - n implants in p bulk 'n in p'
- Many reasons to chose one or the other
  - Polarity of collected carriers (electrons / holes)
  - Availability of material, ease of production (cost)
  - Radiation effects

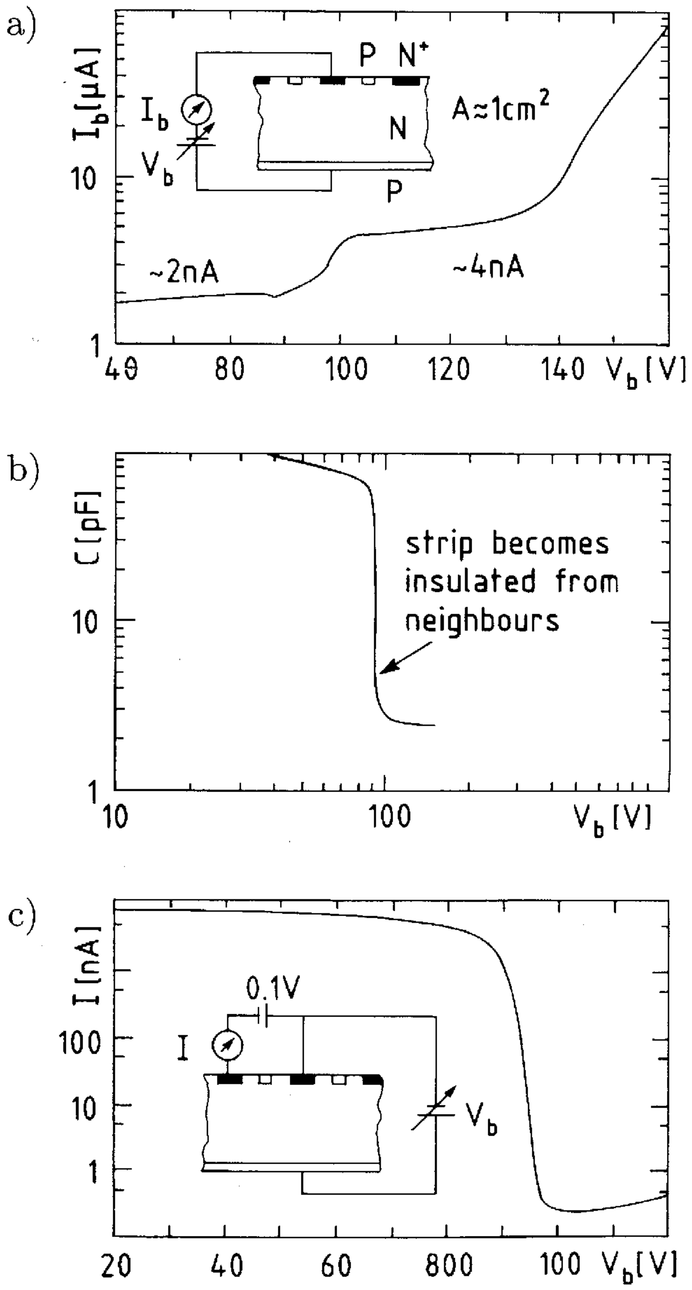
- Watch
  - Leakage
  - Surface leakage
  - Light sensitivity!

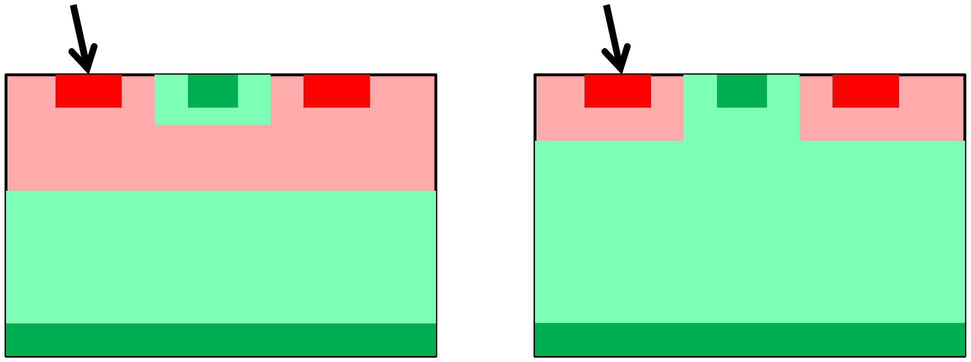
Outlook: Double sided detector (n doped bulk)

- Strips on the ‘ohmic’ n-side **must be isolated** by ‘tricks’
  - Here: shallow p-implant
- Also shown: Guard rings to ‘bring down’ p-side potential



Full depletion with n+ in n strips





< 100 V

> 100 V

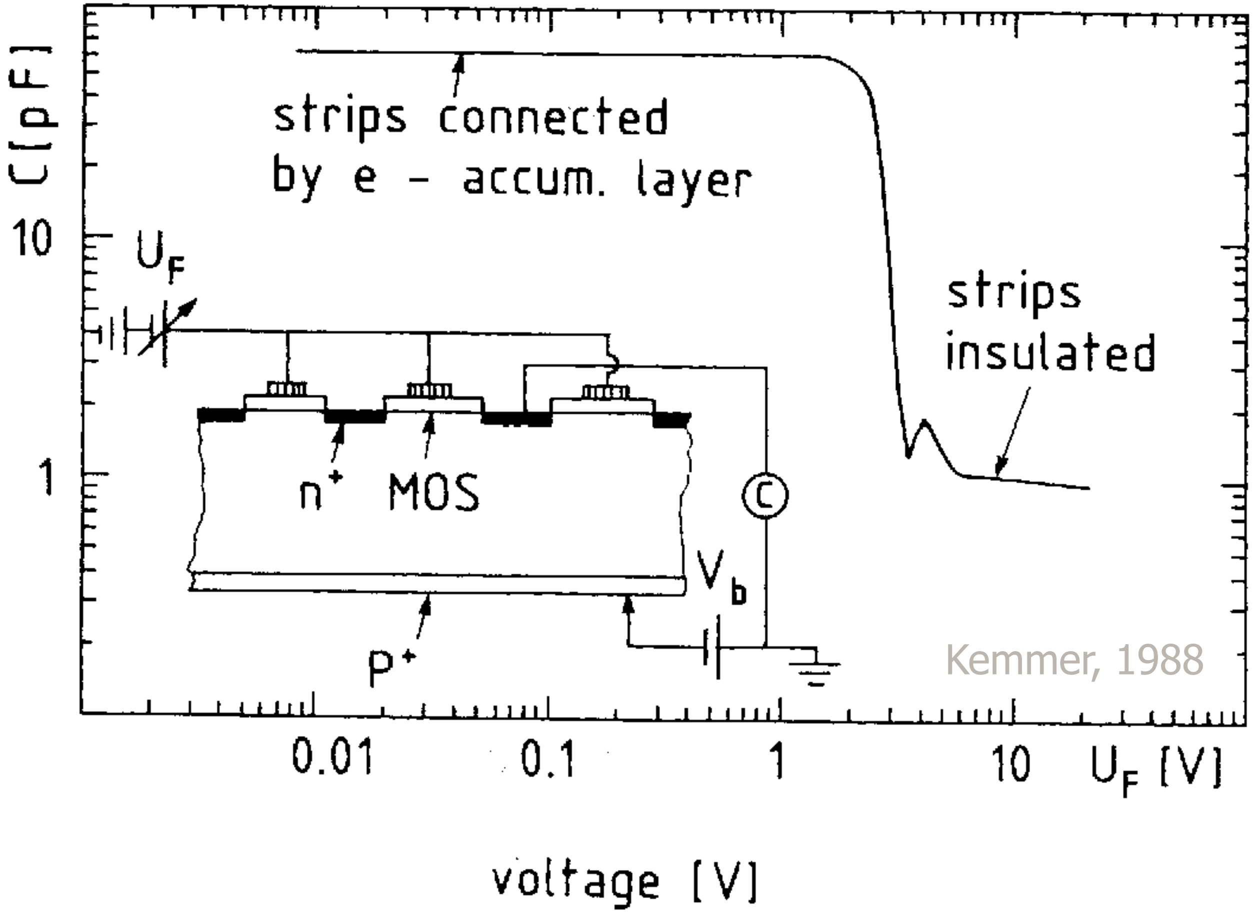
Capacitance of n-strip for increasing bias: At low bias, all strips are shorted → cap is high

Current (resistivity) between 2 n-strips for increasing bias

Kemmer, 1988

Field Plate Isolation between n+ strips

- Can use a MOS-structure (field plate) to isolate the strips:
  - Often heard: ‘electron accumulation layer’ – can be considered as an MNOS which is turned on.



Kemmer, 1988

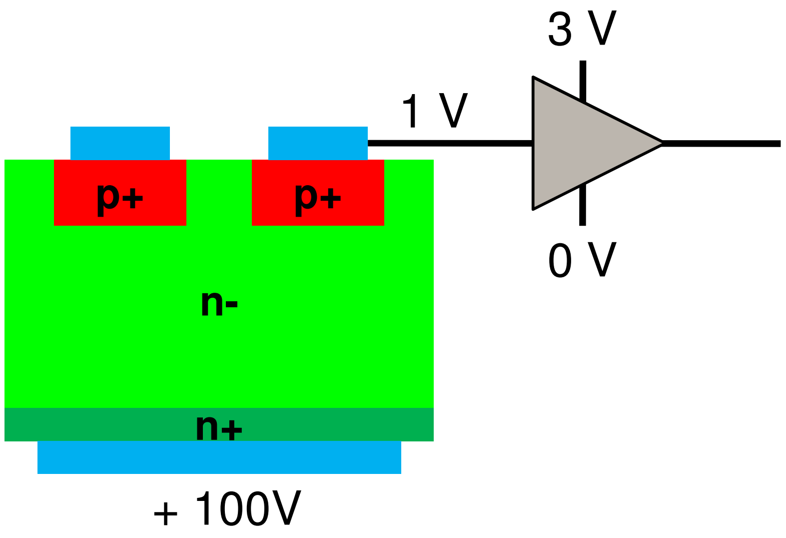
DC coupled detectors

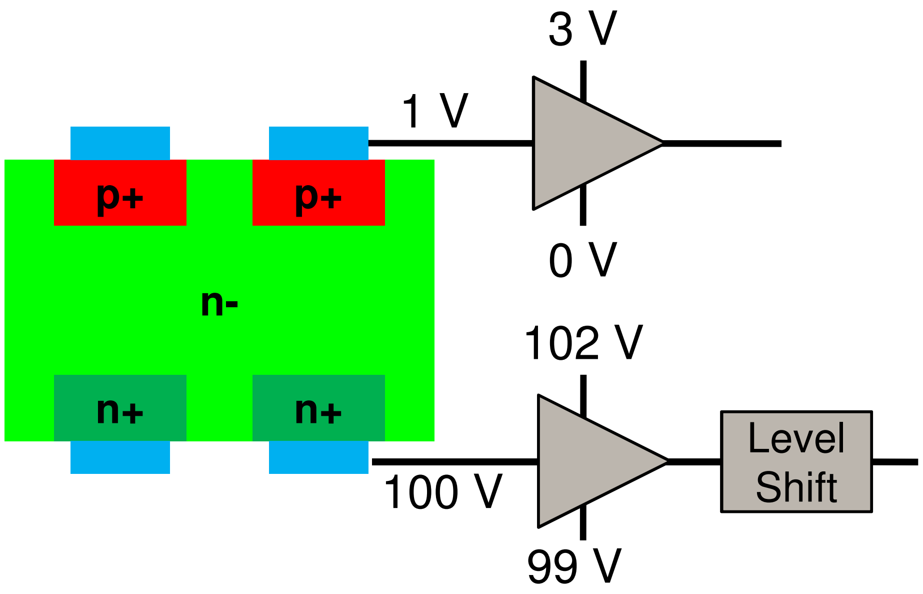
- Strip / pixel is directly connected to amplifier
- Amplifier input defines strip potential

– Amplifier must be on high voltage for double sided detectors. Need level shifters etc.

– Leakage current flows into amplifier.
‘Leakage compensation circuit’ may be required

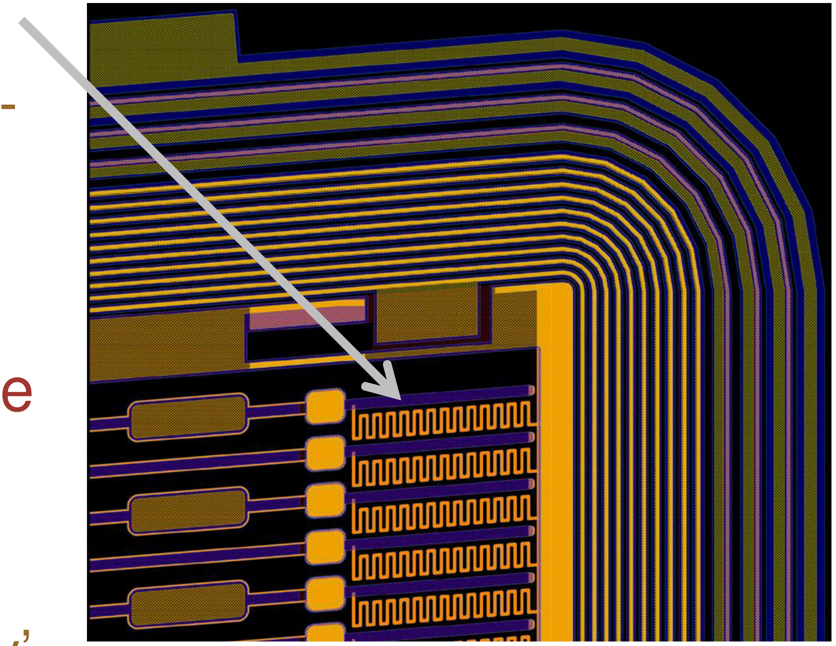
+ Detector is much simpler & cheaper





AC coupled detectors

- Capacitor between strip and amplifier
  - capacitor chip
  - integrated on sensor (isolator between implant and cond. strip)
- Need a ‘bias’ mechanism for strip (to define potential)
  - Polysilicon resistor (high value)
  - ‘punch through bias’ with same-type implant in vicinity ‘pnp’ This can have ‘excess’ noise if current flows!



\+ Amplifier does not see leakage

\+ Amplifier can be at ground

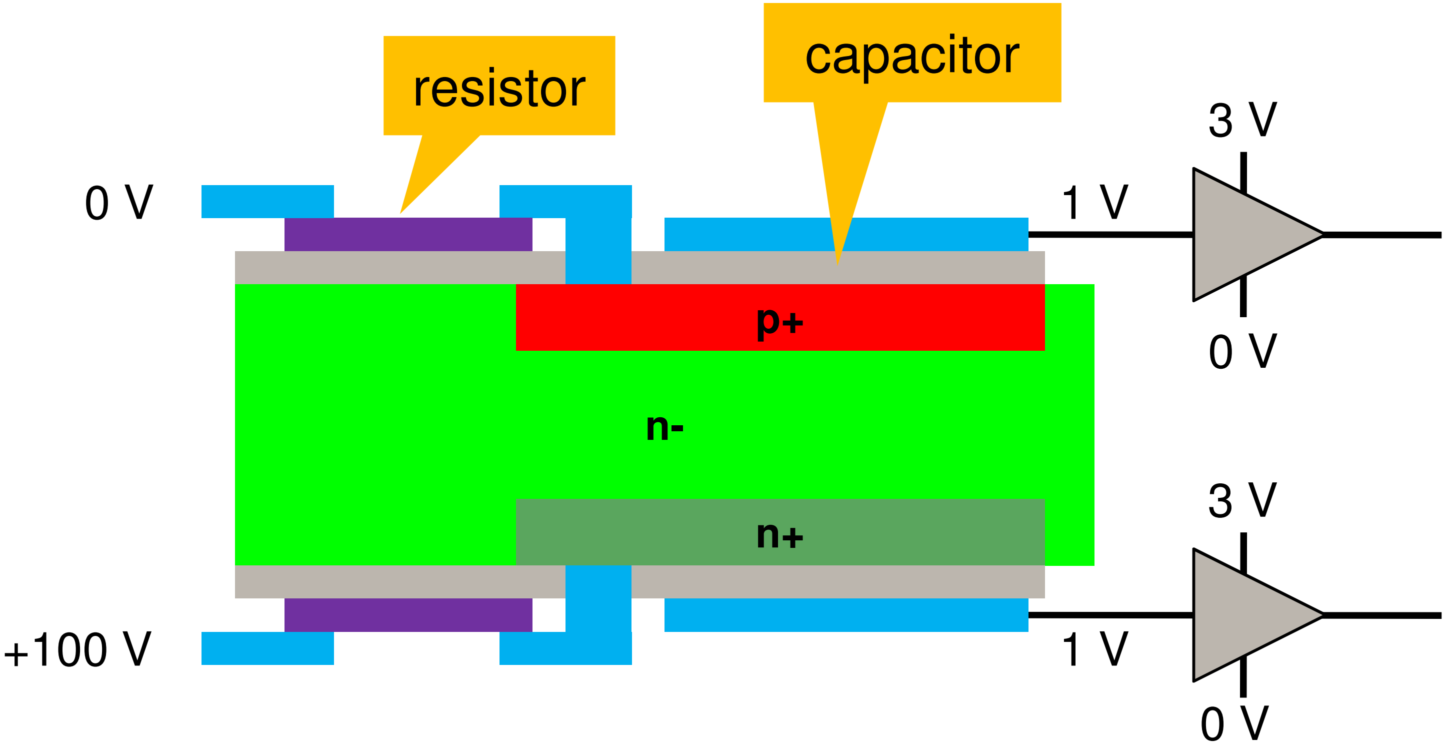
– Broken caps are big problem!

- They pull the strip to a ‘far away’ potential.

– Detector more complicated and more expensive

AC coupled detectors

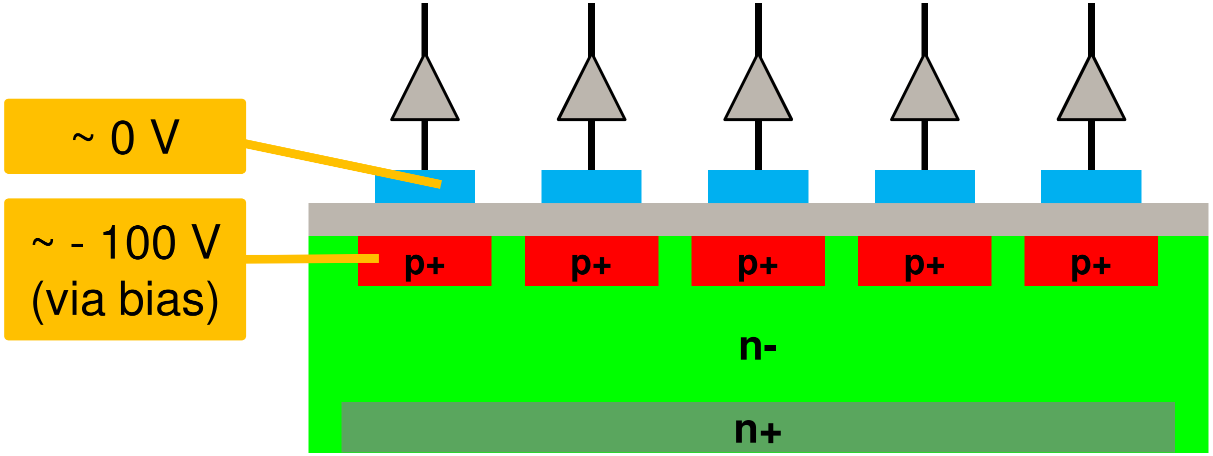
- Double sided with polysilicon bias, side view



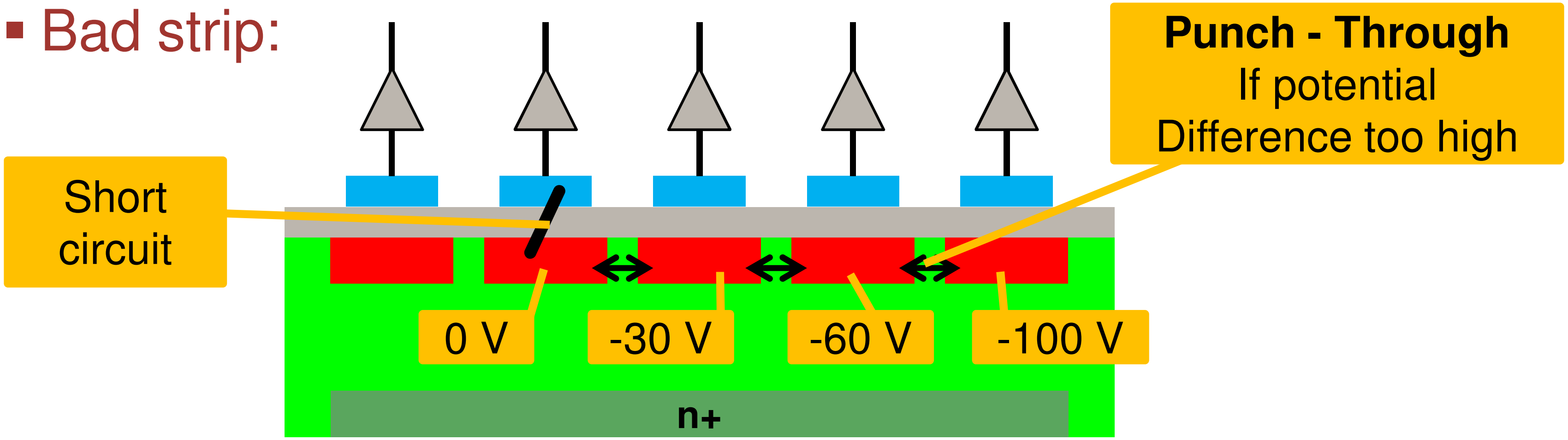
+50 / -50, defekte Strips, region defekt

Problem of Bad Strips in AC Readout

- Cross Section of AC coupled Strips:



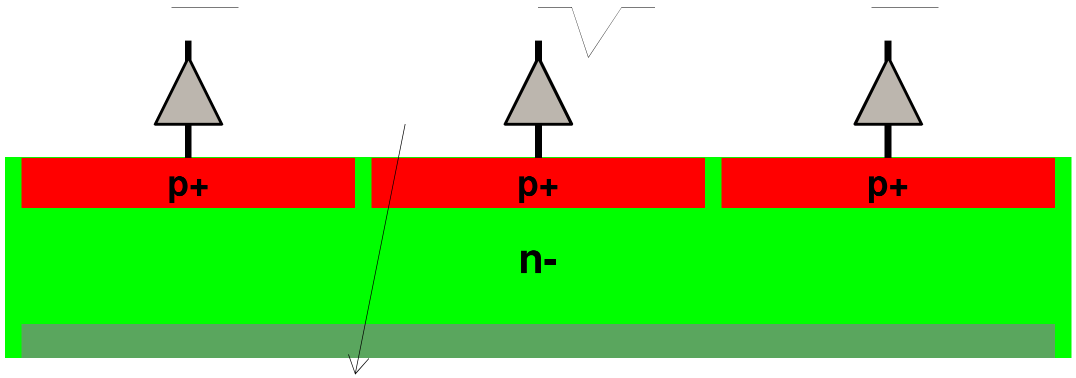
- Bad strip:



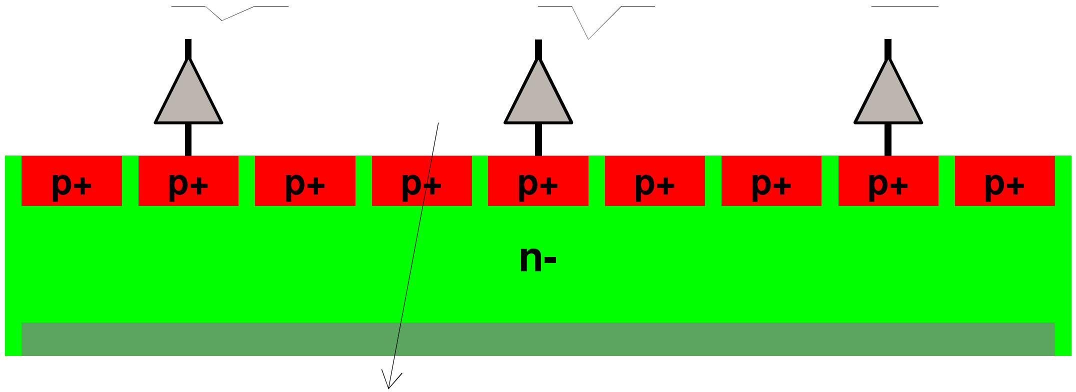
- Several strips are ‘dead’ because their potential drops

Better Position Resolution with Intermediate Strips

- Absorbed particles only deposit charge *locally* (on one strip)
  - → poor position reconstruction (σ = pitch / √12)



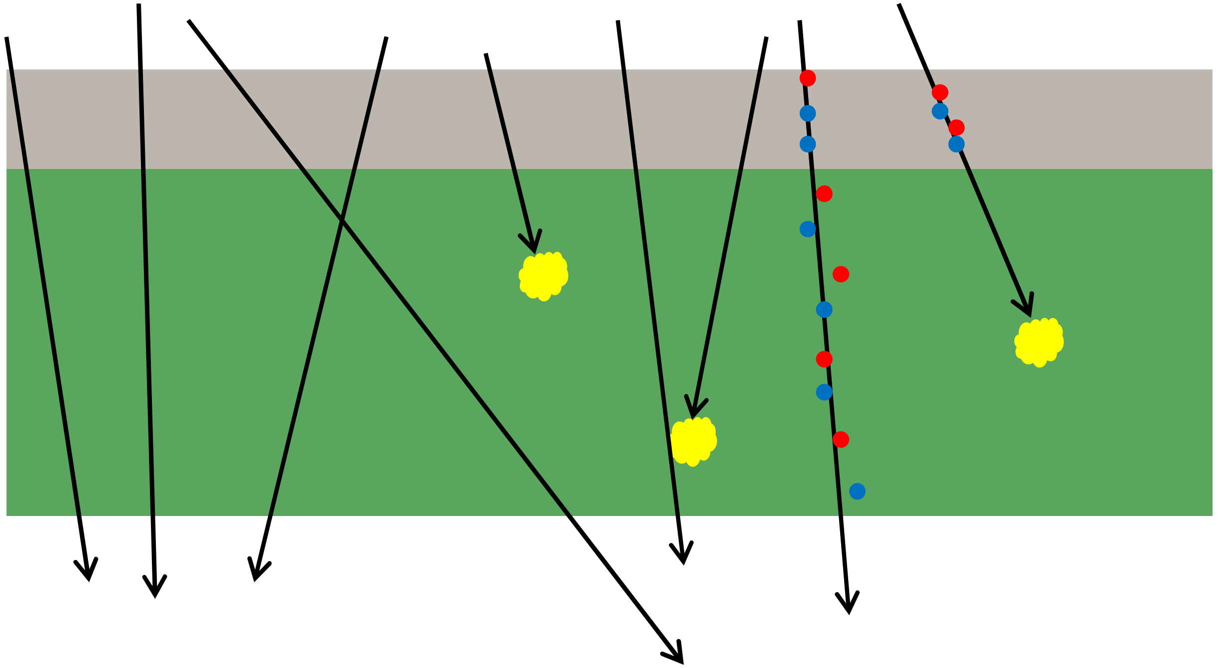
- Can add unconnected (but biased) intermediate strips
  - Signals on these strips share capacitively to neighbours
  - → better position reconstruction by interpolation



RADIATION DAMAGE

Reasons

- Damage by charged particles (same as signal), depositing charges (**T**otal **I**onizing **D**ose, **TID**, given in **Mrad**)
  - No problem in (conducting) bulk (charges are removed)
  - Problem in **Oxide**: e^- are mobile & disappear, holes are stuck
- Nuclear reactions of heavy particles (protons, neutrons): **N**on-**I**onizing **E**nergy **L**oss, **NIEL**, given in $\mathbf{n_{eq}/cm^2}$
  - Atomic structure (crystal) is modified

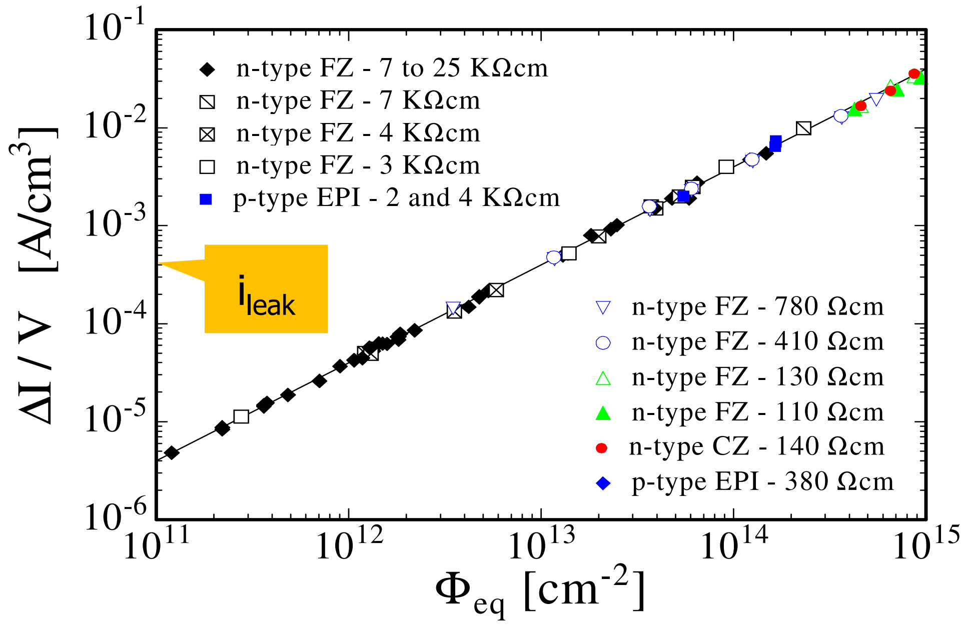


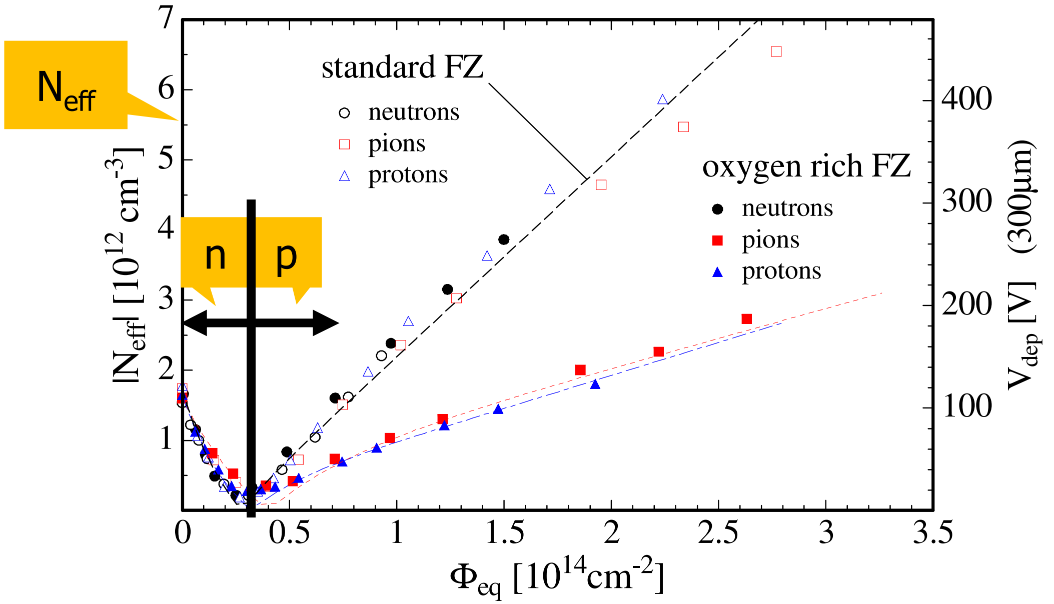
Effects of TID

- Increasing positive oxide charge.
- Threshold of any FET structure shifts (NMOS turn on), parasitic FETs can turn on
- Oxide charges can lead to high field strengths and breakdown → leakage. Careful design needed. All surface potentials must be well defined.

Bulk Damage by NIEL

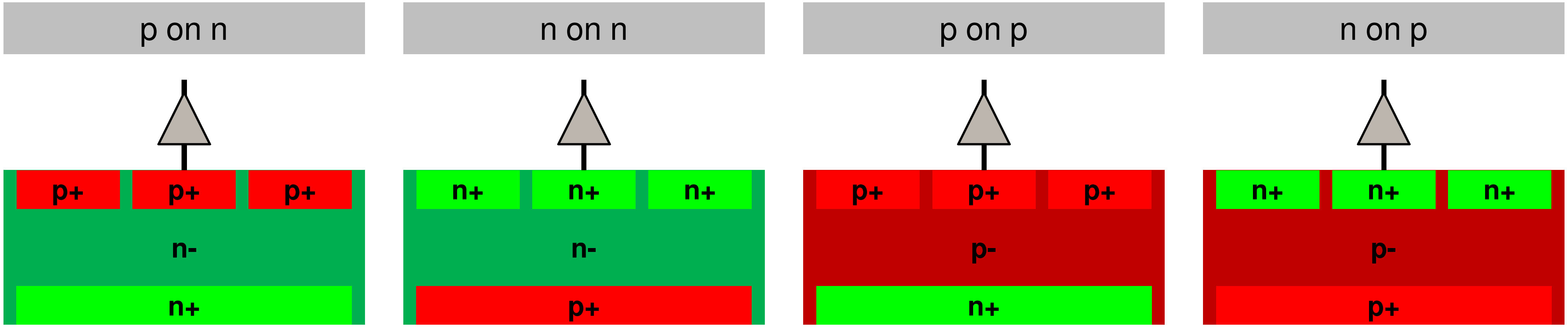
- Defects can act as dopants (depending on energy level in band), as trapping centers, …
  - increased i_{leak} → increased noise
  - Change in doping → bulk ALWAYS becomes p-type!
    → 'Type inversion'
  - Increase in (p-)doping → high depletion voltage, partial depletion
  - Trapping Centers → charges do not reach electrodes





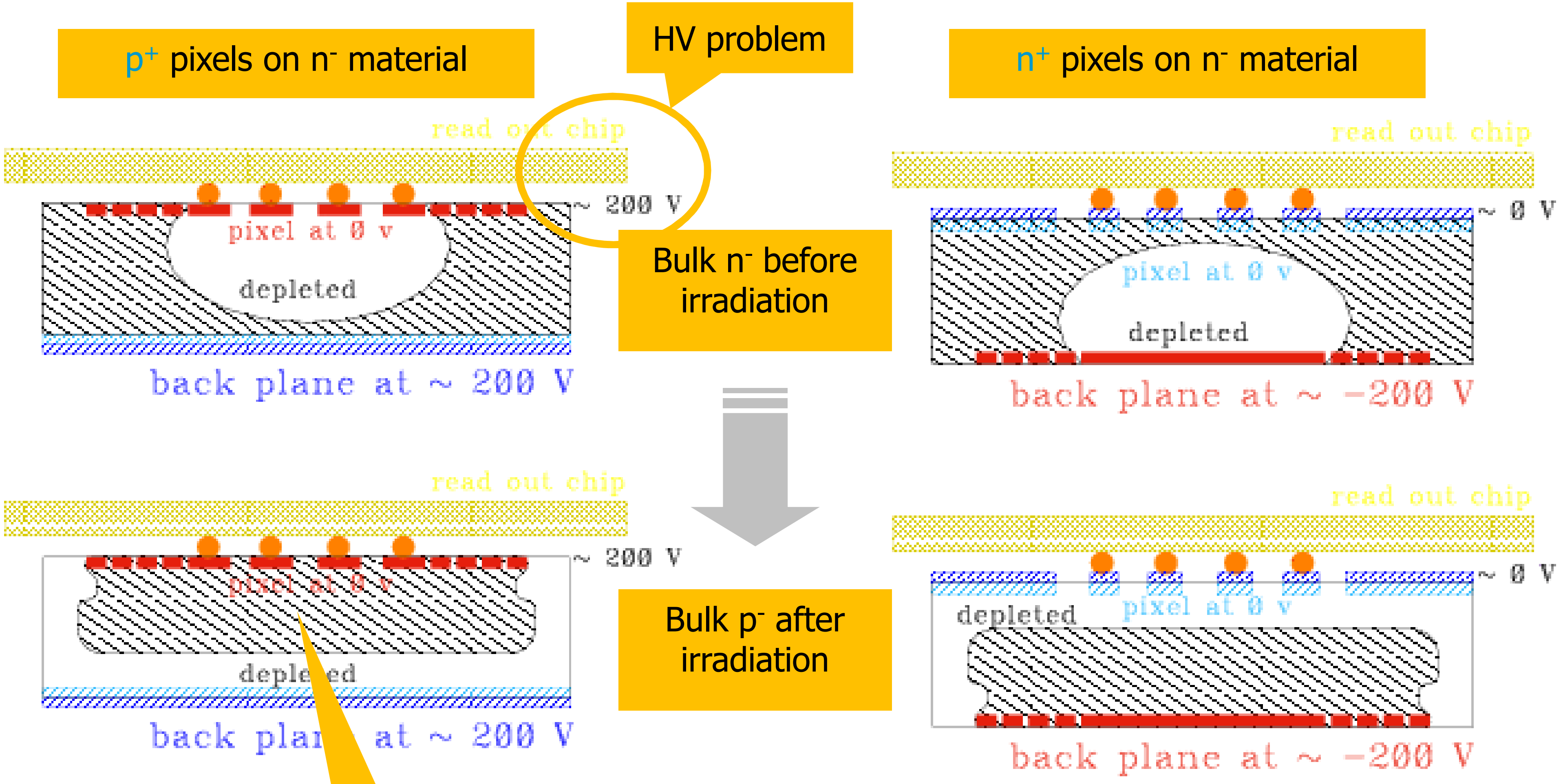
Choice of N vs. P

- For single sided detectors, 4 combinations are possible:



p on n	n on n	p on p	n on p
Irradiation → Type inversion + lower bias voltage		Irradiation immediately increases bulk doping - higher bias voltage	
Hole Collection	Electron Collection	Hole Collection	Electron Collection
Readout Initial: Diode Later: Ohmic	Readout Initial: Ohmic Later: Diode	Readout Initial: Ohmic Later: Ohmic	Readout Initial: Diode Later: Diode
	↑ LHC		

Problem of type inversion (n-bulk)



p^+ pixels on n^- material

HV problem

n^+ pixels on n^- material

Bulk n^- before irradiation

Bulk p^- after irradiation

Need full depletion!

☹

☺

Can be operated partially depleted

Annealing

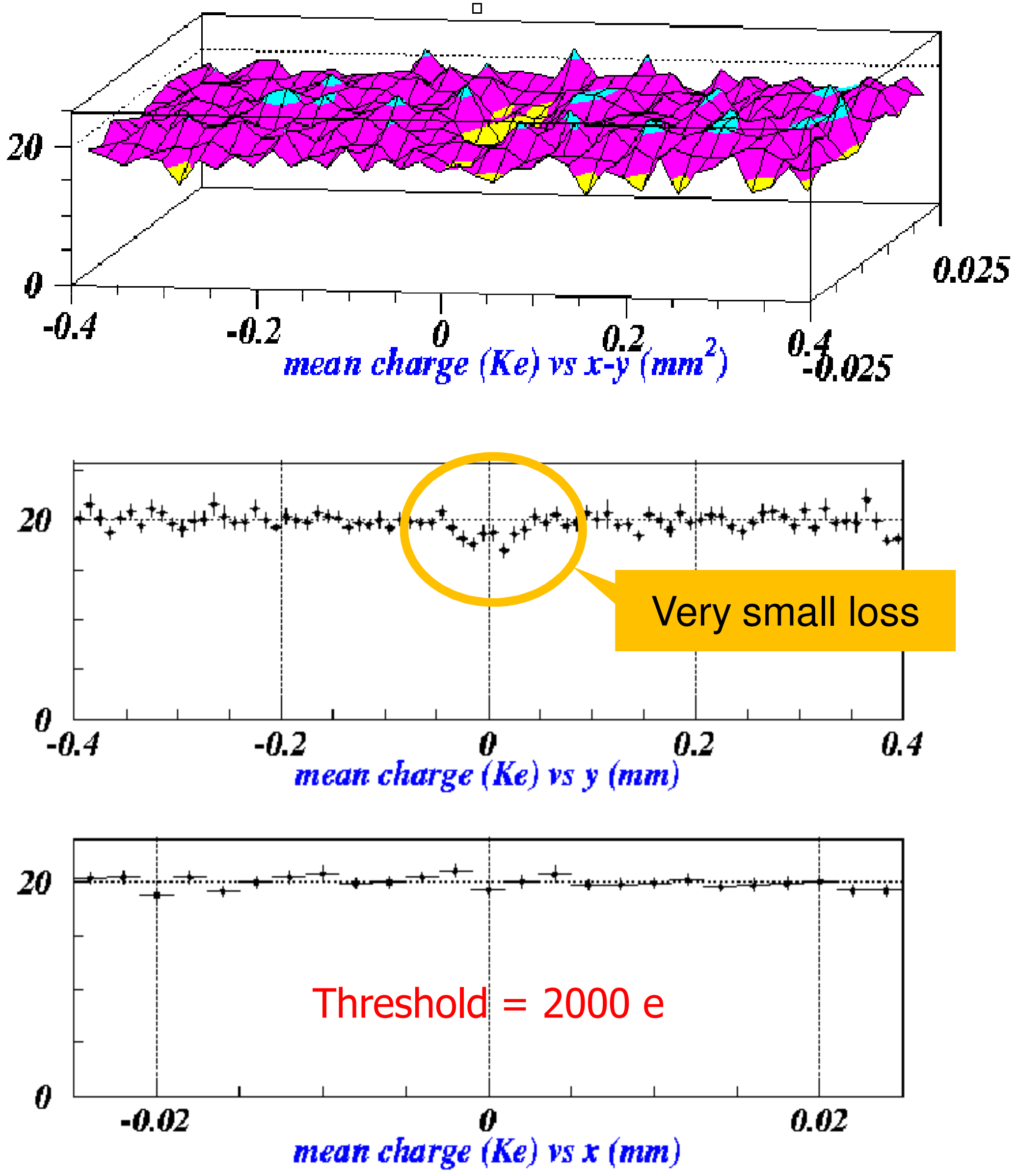
- Some crystal defects (vacancies, atoms at wrong place) can be ‘repaired’ by heating (operating ‘warm’)
  This is called ‘**annealing**’
- On the other hand, harmless defects can convert into ‘bad’ defects: ‘**reverse annealing**’
- Inactive atoms in bulk (Oxygen) can catch away ‘bad’ defects → ‘defect engineering’

- Keeping detectors cold (-6°C) reduces effect on leakage current!
- Using ‘oxygenated’ silicon slows down type inversion

Performance of irradiated (ATLAS) pixel sensor

- Sensors irradiated to full ATLAS fluence (10^{15} n_{eq}/cm^2)
- Test beam with reference detector to get hit position
- Measurement of charge

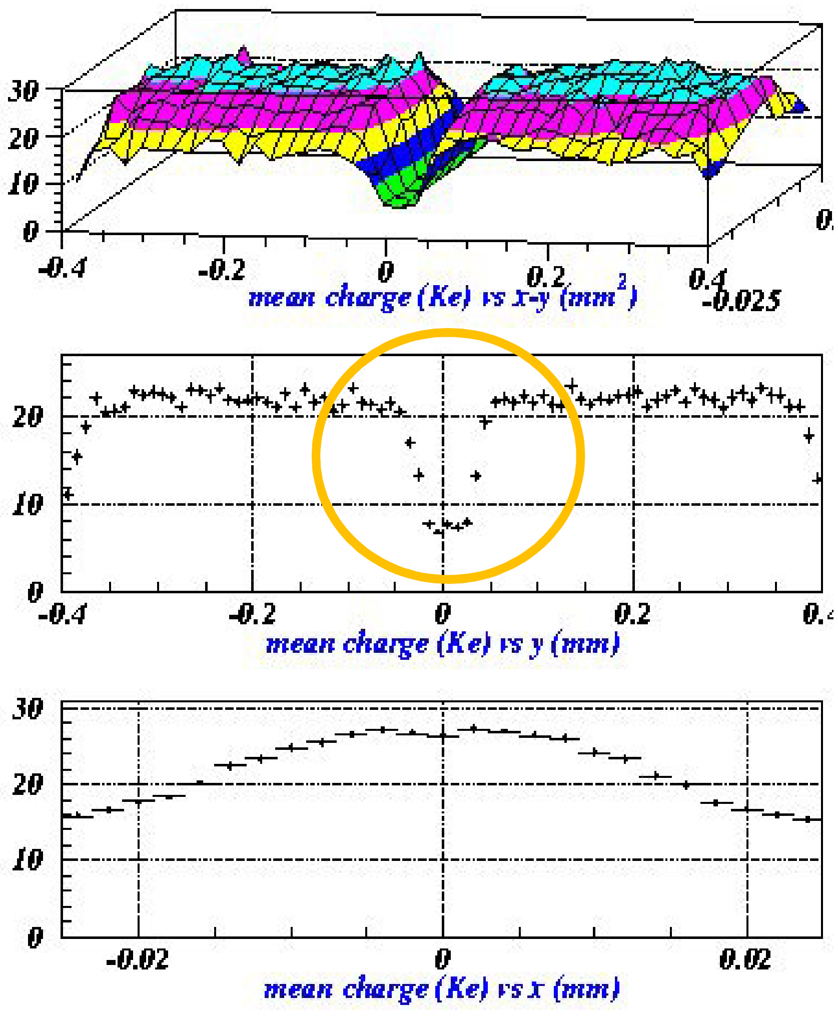
- Homogenous charge collection
  also in pixel corners
- V_{bias} > 600V possible!

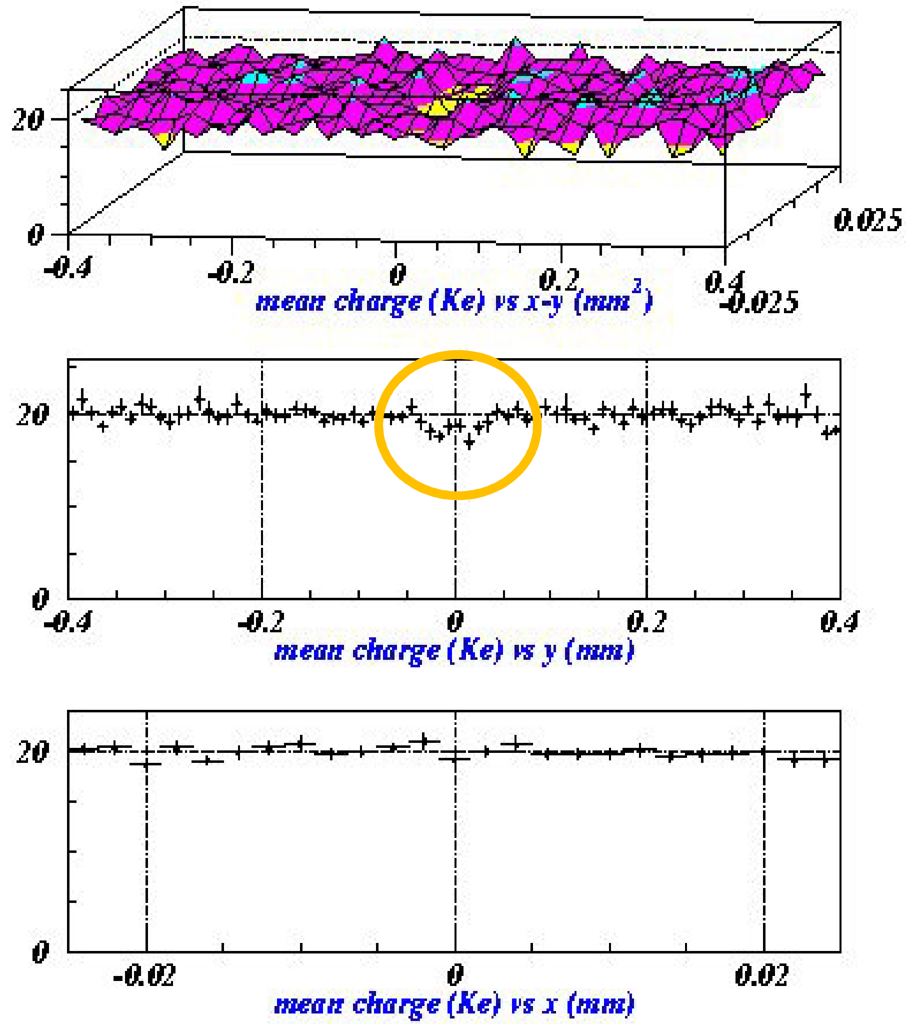


Very small loss

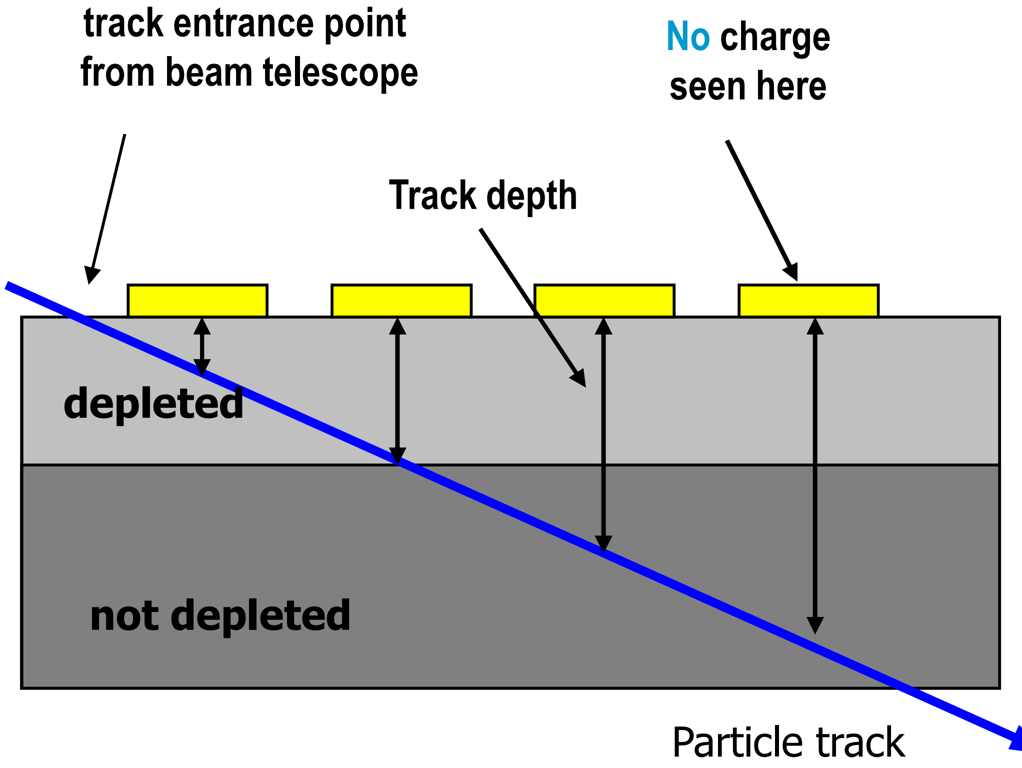
Threshold = 2000 e

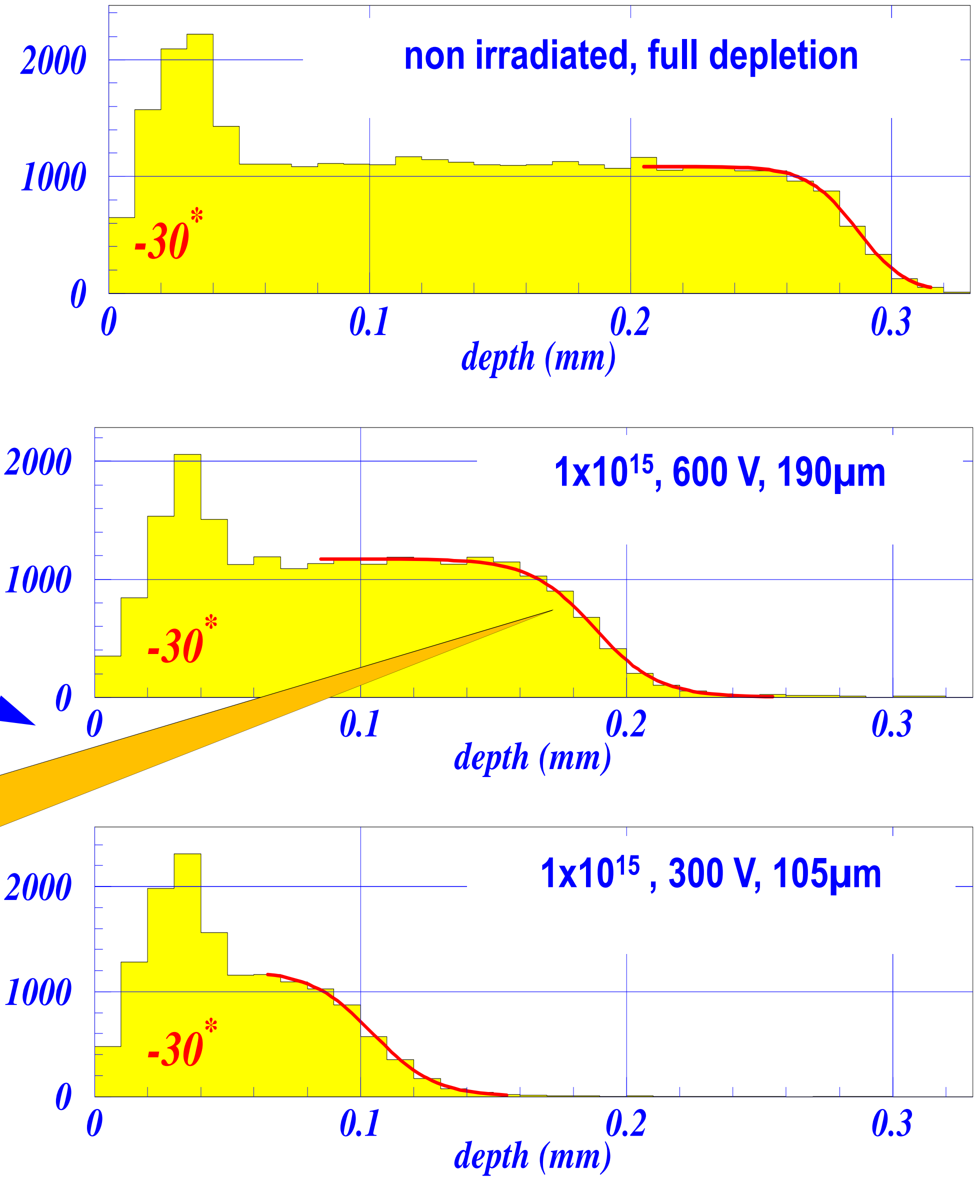
Comparison of two Sensor designs





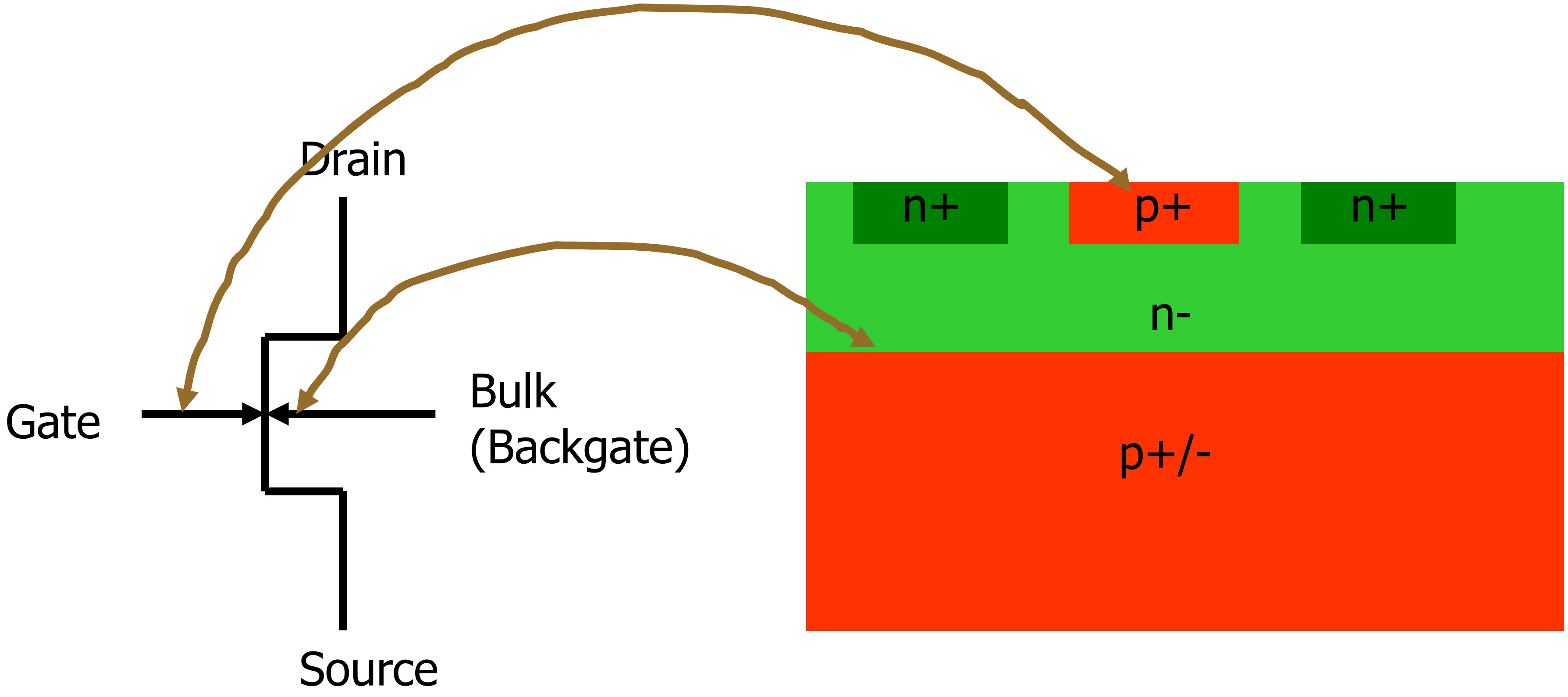
Partial Depletion of ATLAS Pixel Detectors





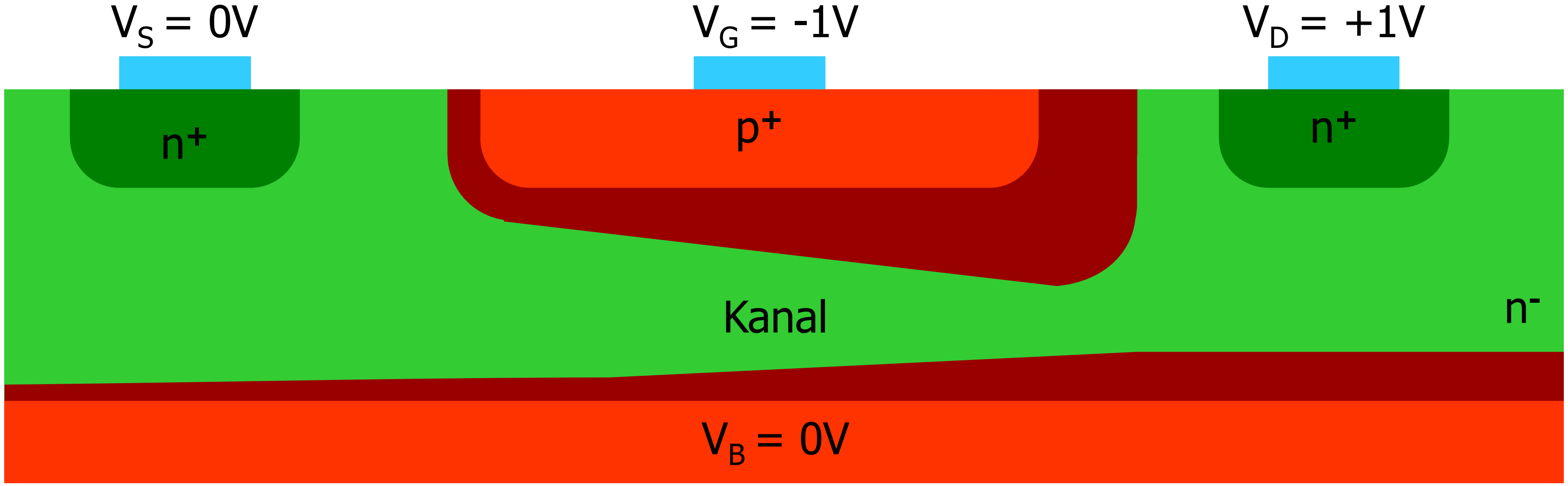
Depletion depth is 190 µm @ 600 V
after 10^{15} cm^{-2} (full ATLAS dose!)

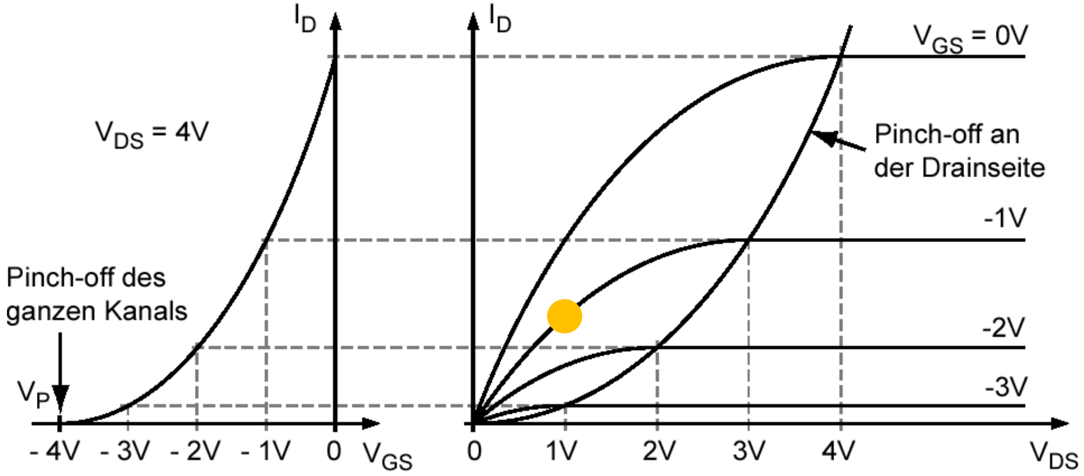
Quick look at the JFET



JFET: channel open, linear region

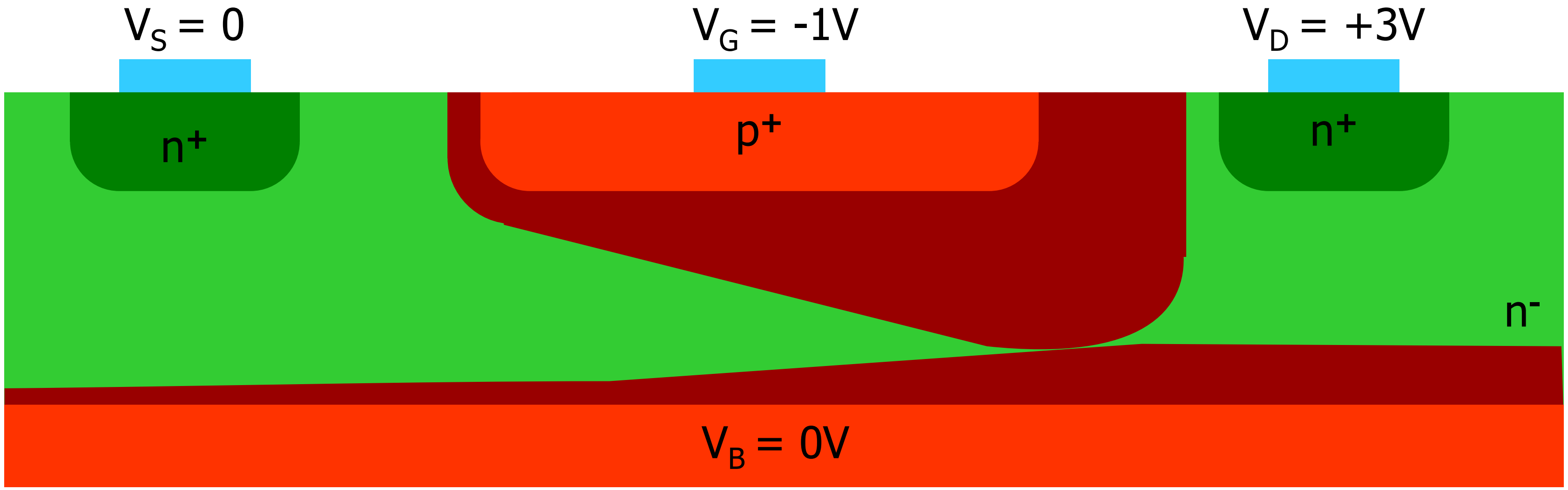
- Channel cross section is reduced by two depletion regions
- In linear region, channel is still open

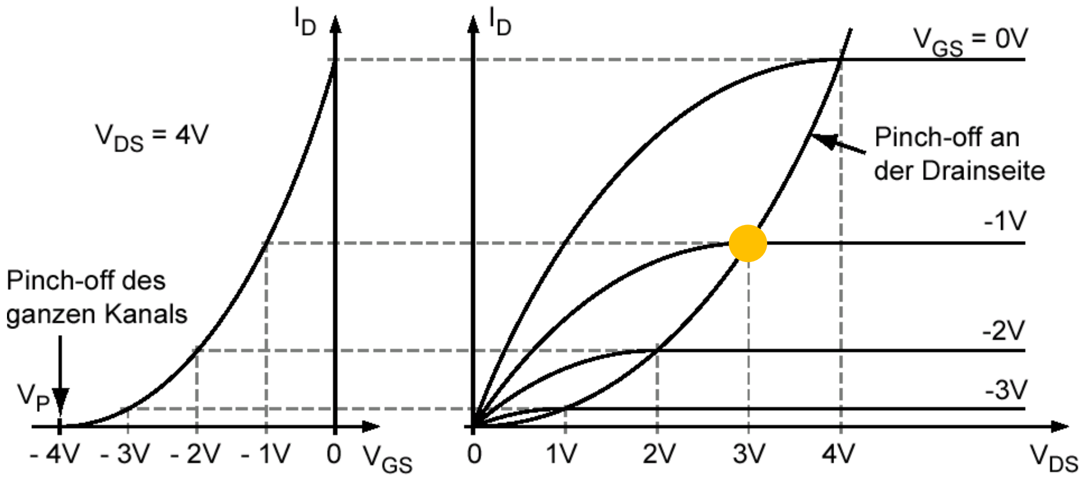




JFET: Saturation

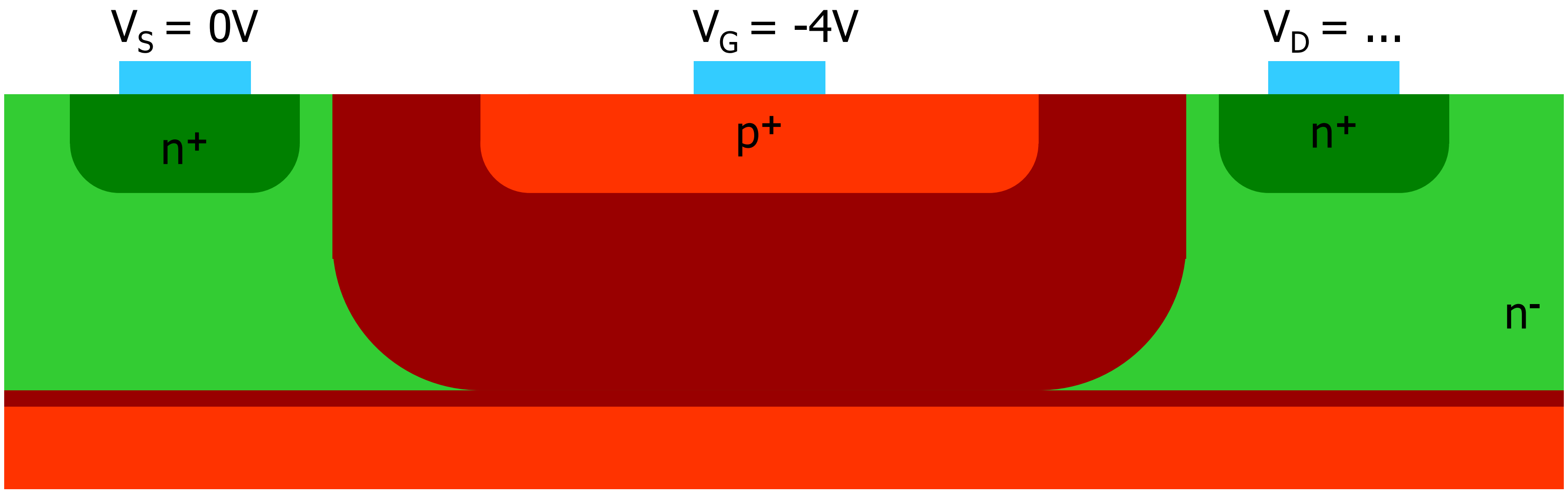
- At high Drain-Gate-Voltage, the drain side is pinched off.
- Further increase in drain voltage does not increase current

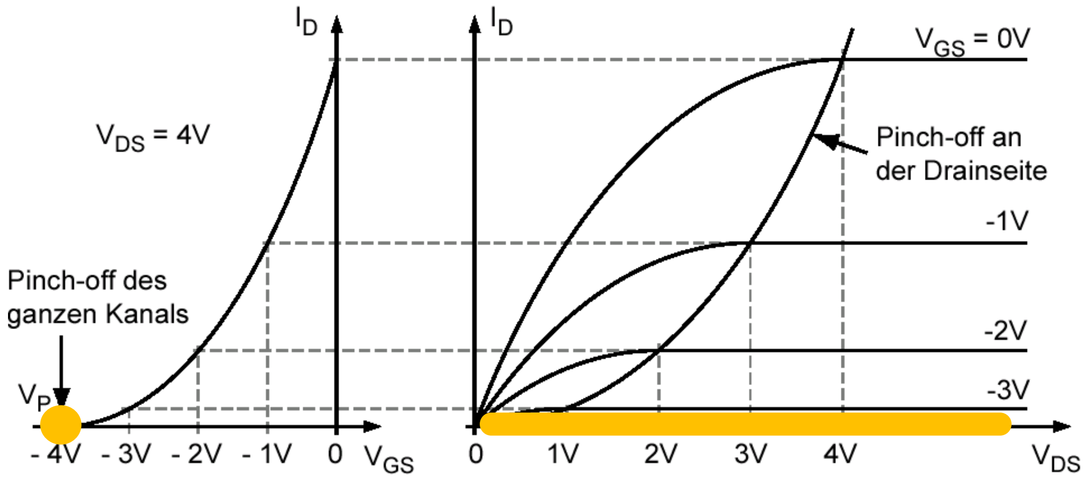




JFET: Complete pinch-off

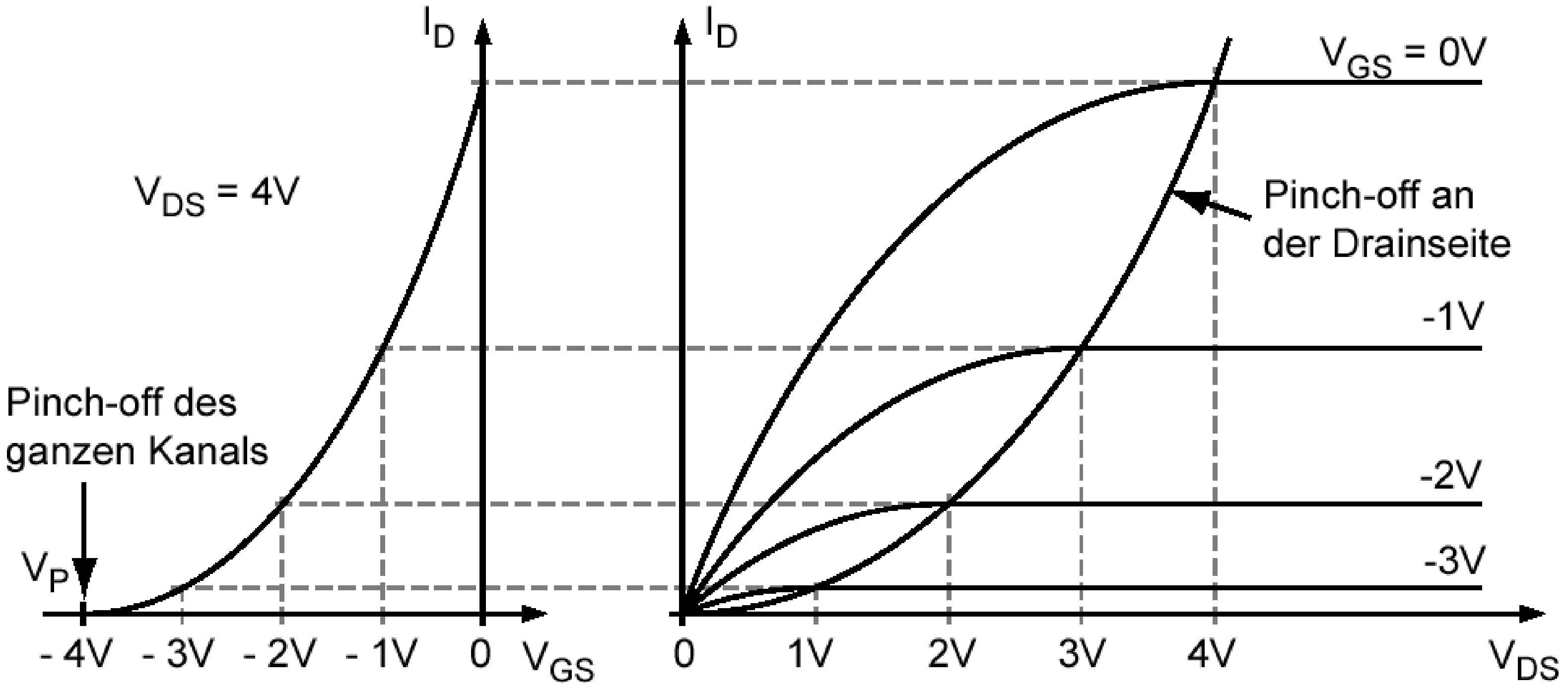
- At negative gate voltage, the channel is pinched off completely
- There is no current at all drain voltages





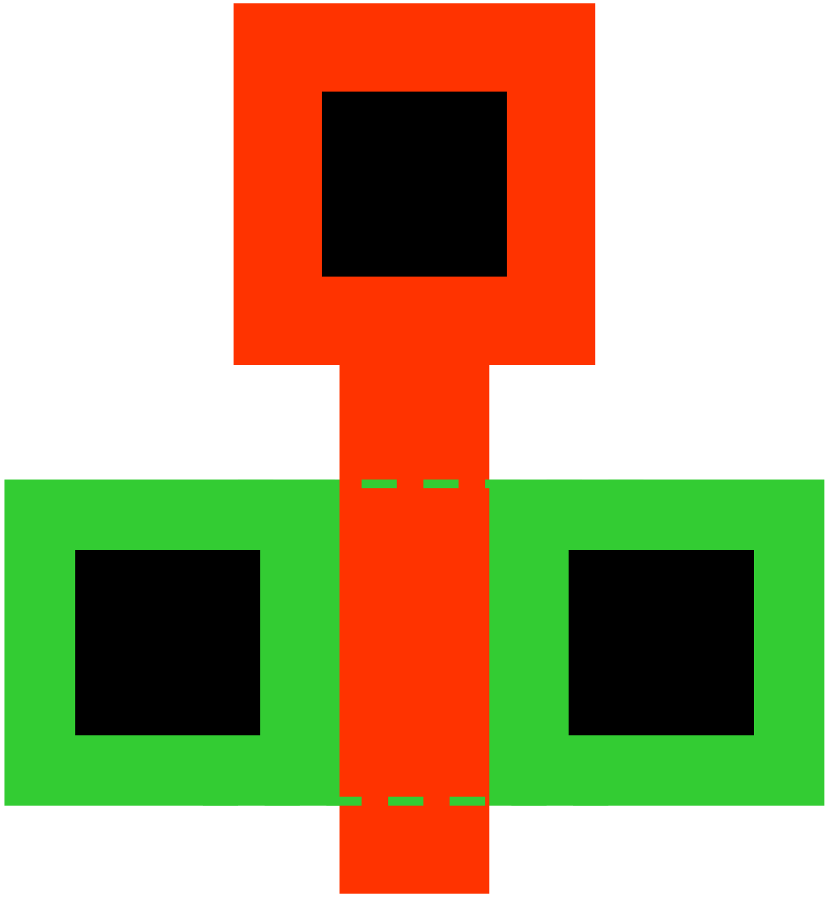
JFET: characteristic

$$I_D = \frac{W}{L} I_0 \left[\frac{V_D}{V_p} + \frac{2}{3}\left(\frac{V_G'}{V_p}\right)^{3/2} - \frac{2}{3}\left(\frac{V_D + V_G'}{V_p}\right)^{3/2} \right] \qquad V_G' := V_{bi} - V_G$$

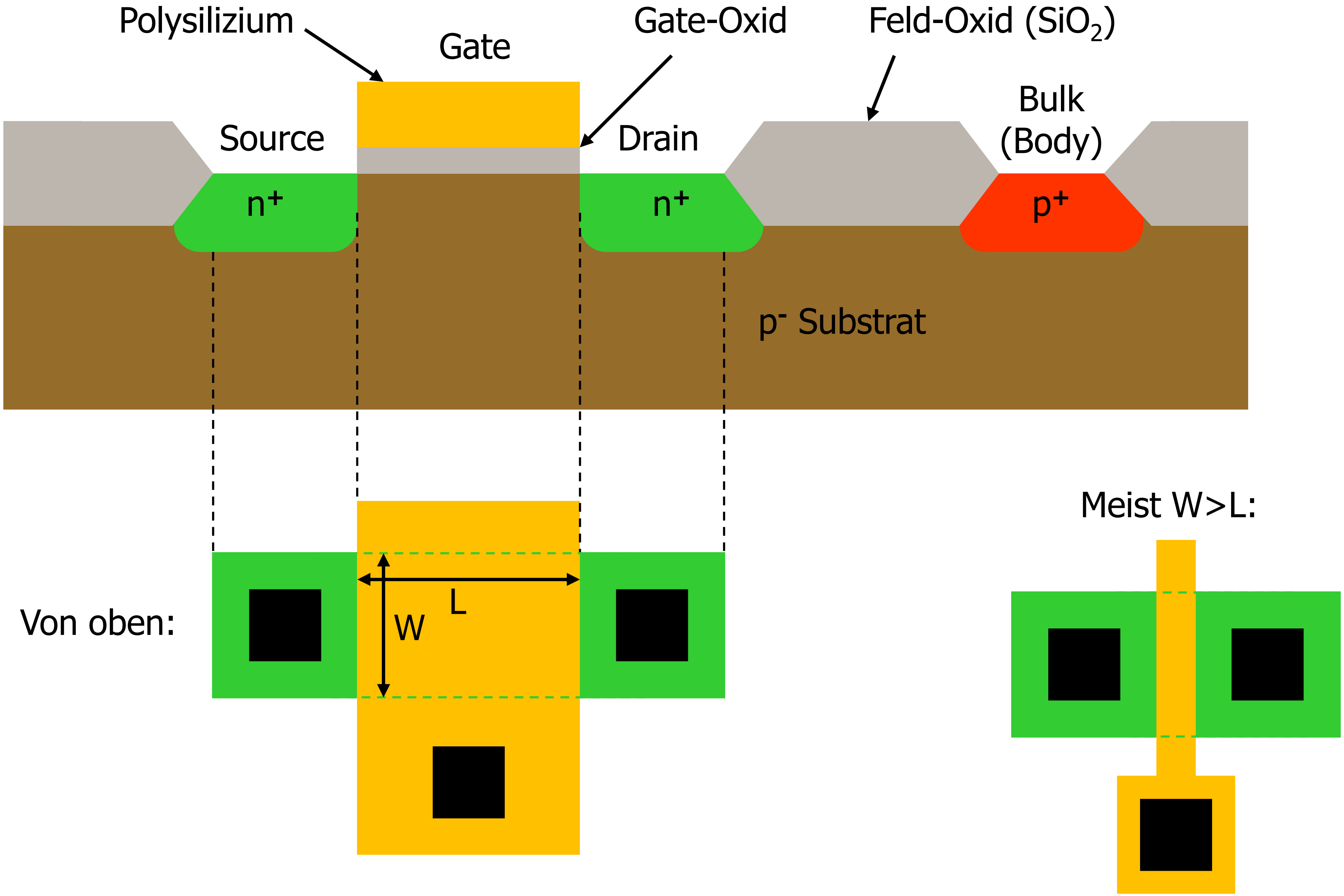


- At high drain voltages, channel becomes shorter and current increases a bit: ‘channel length modulation’ / ‘Early Effect’

The MOS Transistor



Cross Section of a NMOS Transistors



Meist W>L:

MOS: Accumulation – Depletion – Inversion

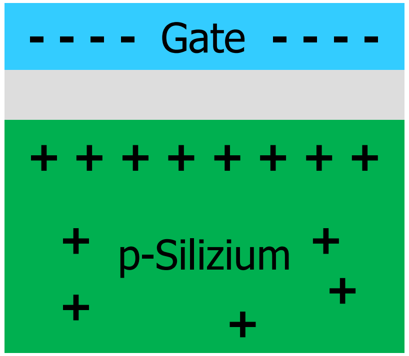
- Consider isolated gate electrode on p-silicon

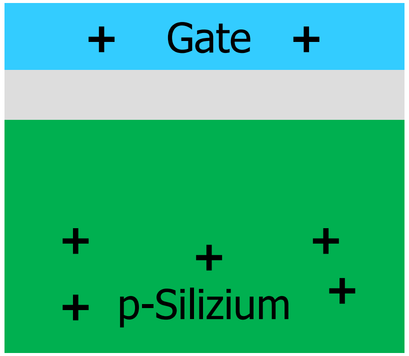
Gate **sehr negativ**:
- reichlich vorhandene Löcher werden unter d. Gate gezogen
- ‚**Akkumulation**`
- ‘Kondensatorplatte’ direkt unter dem Oxid
- Kapazität ist maximal

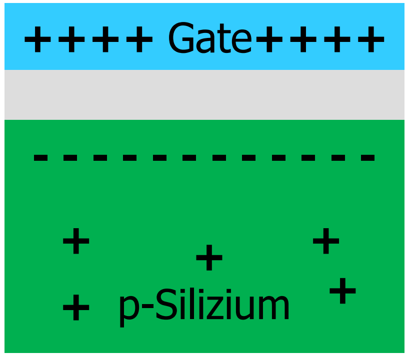
Gate **positiver**:
- Löcher werden weggedrückt

-‚**Verarmung**`
- ‘Kondensatorplatte’ weiter im Bulk (Dichte negativer Raumladung durch Dotierung begrenzt)
- Kapazität sinkt

Gate **sehr positiv**:
- Elektronen (Minoritätsträger) werden angesaugt
- ‚**Inversion**`
- ‘Kondensatorplatte’ wieder direkt unter Oxid
- Kapazität wieder maximal

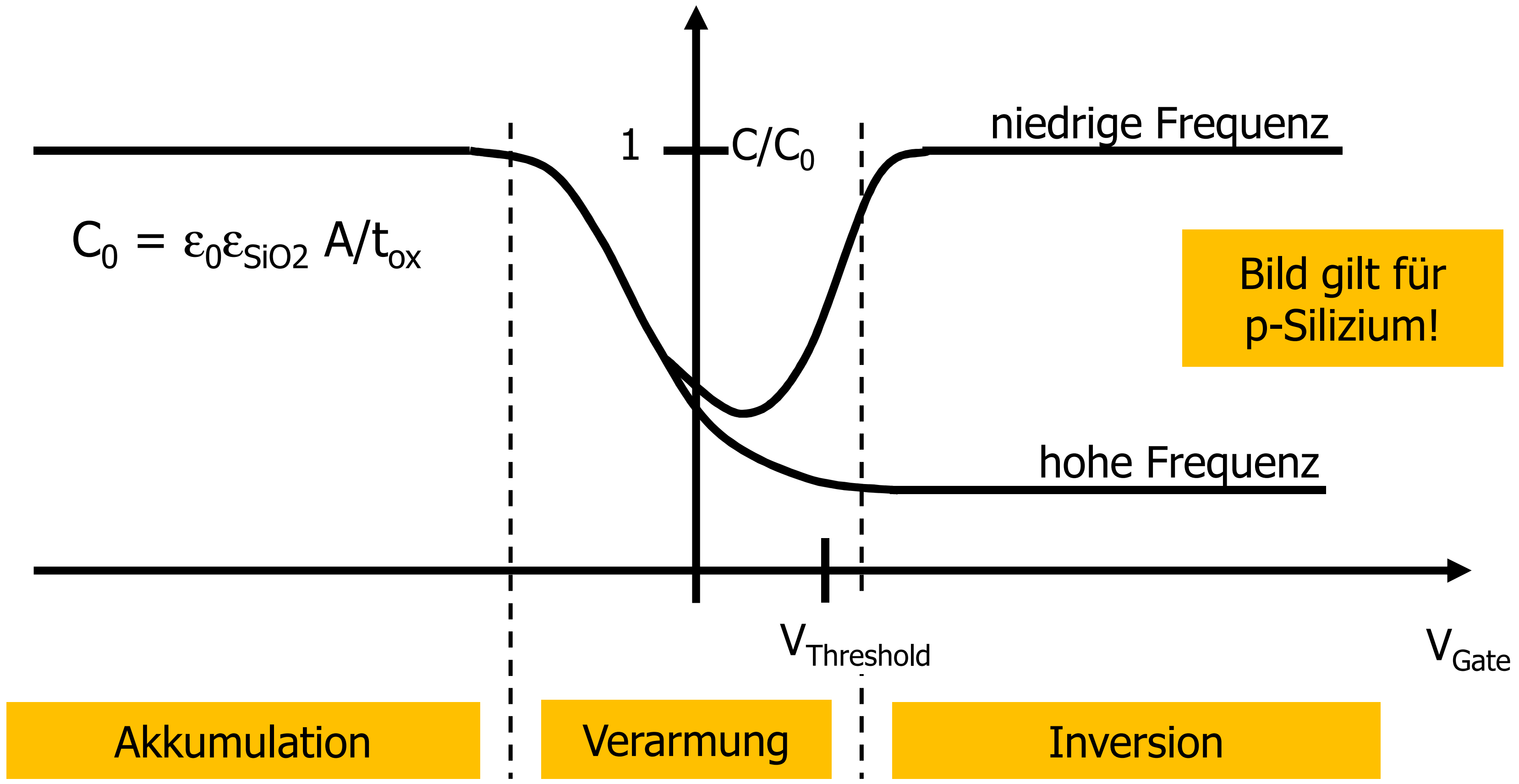






- Siehe Skript und Applet http://smile.unibw-hamburg.de/Bauelemente/FET/Mos_struktur.htm

MOS: Accumulation – Depletion – Inversion



$C_0 = \varepsilon_0 \varepsilon_{SiO2} A/t_{ox}$

Bild gilt für p-Silizium!

- MOS Struktur im Bänderdiagramm:smile.unibw-hamburg.de/Bauelemente/FET/Baender_MOS_struktur.htm
- Bei hohen Messfrequenzen können in Inversion die Elektronen nicht schnell genug angesaugt werden. Die Kapazität bleibt dann klein.
- Man definiert die Schwellenspannung (für 'starke' Inversion) oft als die Spannung, bei der die Elektronendichte in der Inversionsschicht so groß ist wie die Löcherdichte im Bulk.

MOS in band diagram

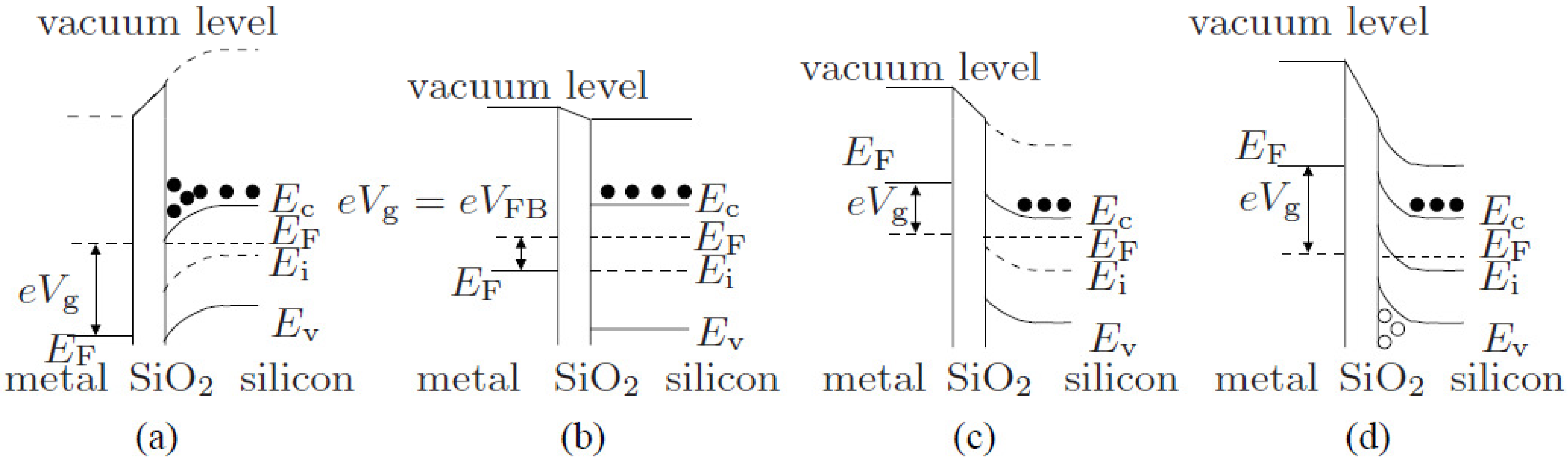
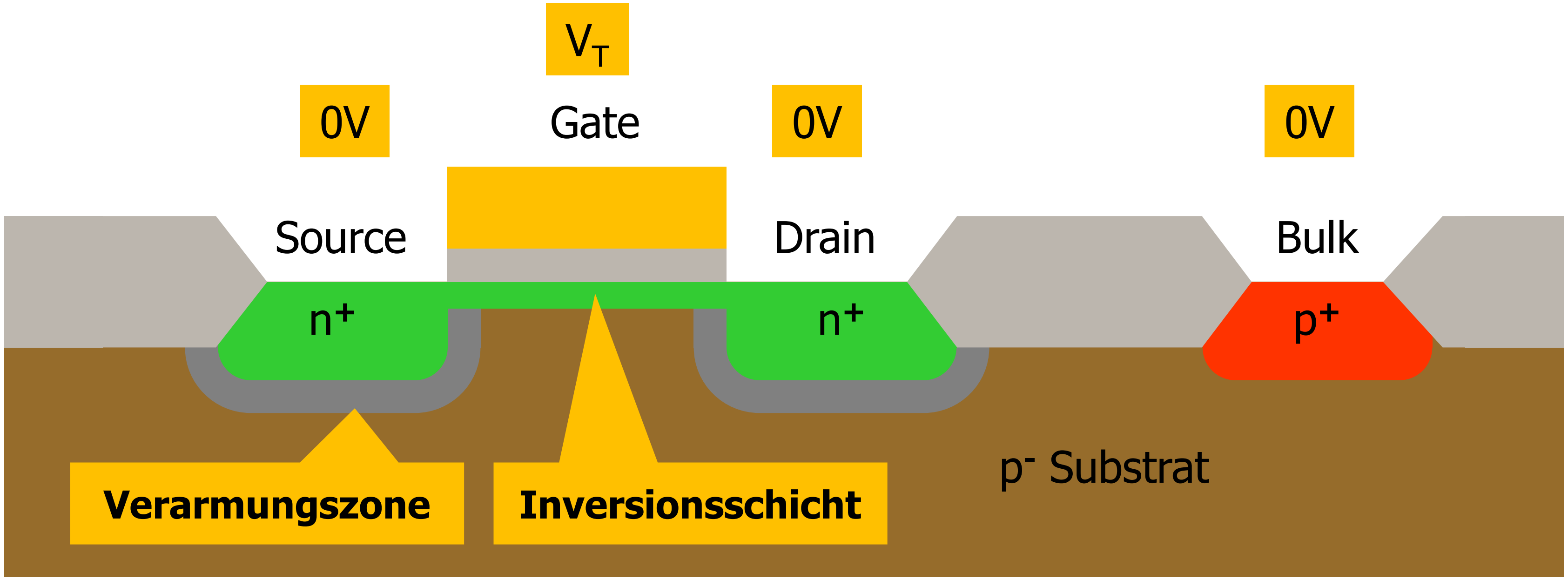


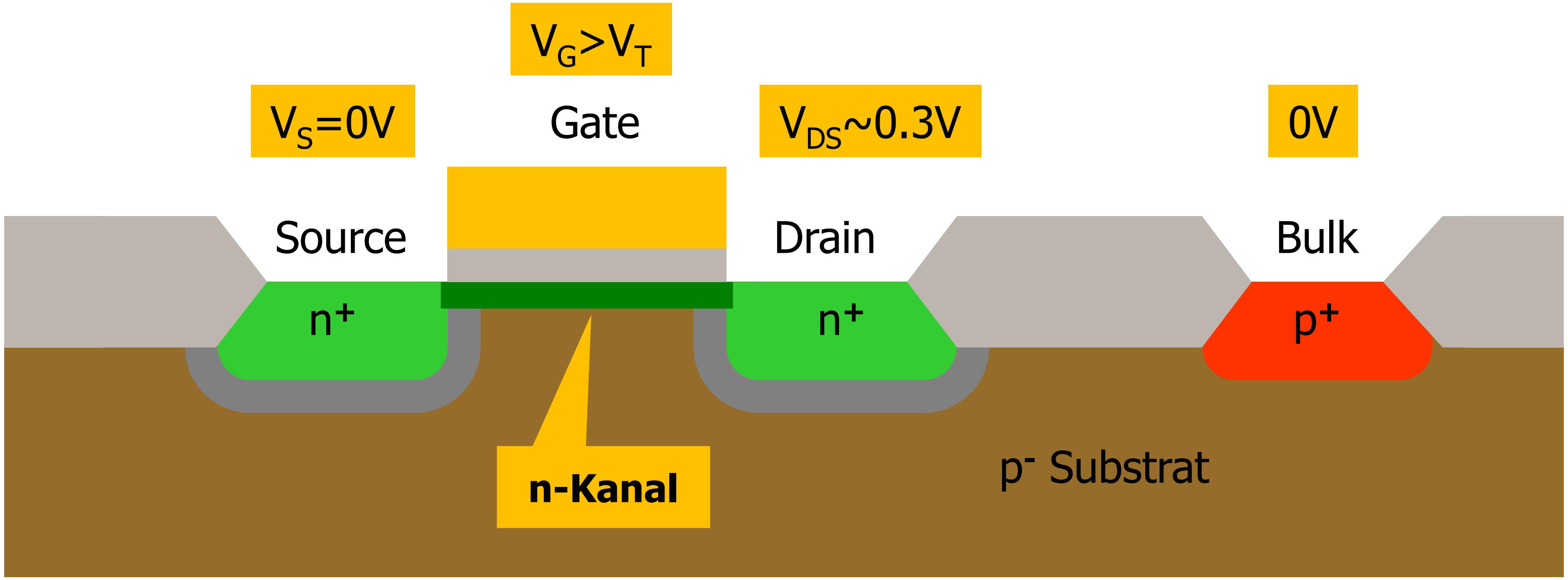
Fig. 2.10. Band diagram of a MOS structure in accumulation (a), flat band condition (b), depletion (c), and inversion (d). The filled circles (•) indicate electrons, the open (○) holes

Threshold Voltage



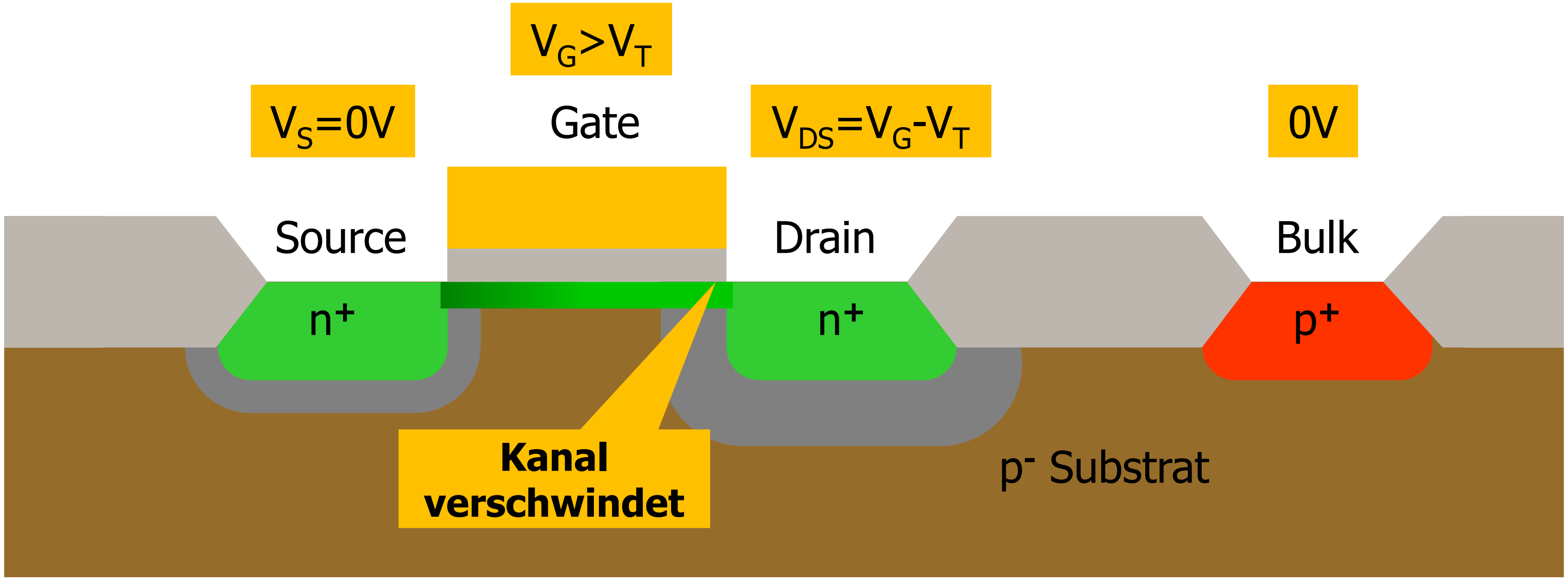
- An den Drain- und Source-Dioden bilden sich **Verarmungszonen** aus
- Bei genügend positivem Gate bildet sich unter dem Gate eine (n-leitende) **Inversionsschicht** aus, durch die Strom von Drain nach Source fließen kann.
- Die **Gate-Source-Spannung V_{GS}**, ab der starke Inversion vorliegt, ist die **Schwellenspannung V_T**

Linear Region



- Bei kleinen Drain-Source-Spannungen V_{DS} bleibt der **Kanal** erhalten
- Dies ist der Fall solange **V_{DS} < V_{GS}-V_T**
- Der Transistor verhält sich wie ein Widerstand, der bei $V_{GS} < V_T$ unendlich wird: **$I_D = a \times V_{DS}$**
- Man spricht vom **Linearen Bereich**

Saturation



- Bei sehr positiver Drain-Spannung verschwindet der Kanal an der Drain-Seite.
- Man spricht von '**pinch-off**' (Abschnüren)
- Diese **Sättigung** tritt ein, wenn $\mathbf{V_{DS} = V_{DSat} = V_{GS}-V_T}$
- **Der Strom steigt mit steigendem $\mathbf{V_{DS} > V_{GS}-V_T}$** (fast) **nicht weiter an**
- Genauer: Da mit steigendem V_{DS} die Länge des Kanals abnimmt, steigt der Strom weiter leicht an. Man spricht von **Kanallängenmodulation**

Strom-Spannungs-Formeln in starker Inversion

- Formeln in 'starker Inversion', d.h. VGS >> VT

$$I_D = K_N \frac{W}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right] \quad \text{für} \quad V_{DS} < V_{GS} - V_T$$

- Linearer Bereich:

$$I_D = \frac{K_N}{2}\frac{W}{L}(V_{GS} - V_T)^2(1 + \lambda V_{DS}) \quad \text{für} \quad V_{DS} > V_{GS} - V_T$$

- Sättigung:

$$K_N = \mu_N C_{OX} = \mu_N \frac{\varepsilon_0 \varepsilon_{OX}}{t_{OX}}$$

t_{ox} = Oxid-Dicke

- Daneben gibt es auch den Bereich **schwacher Inversion**, **'weak Inversion'** oder **Subthreshold**-Bereich. Dort ist der Drainstrom klein und hängt exponentiell von V_{GS} ab